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Lower slenderness limits for unbraced and transversely loaded braced r.c. compression members.

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ABSTRACT

Effects of axial forces on the displacements in compression members, generally termed second order effects, are small and can be neglected in a great many structures. In recognition of this, most design codes and standards for reinforced concrete structures give lower slenderness limits for compression members, and allow structures to be designed for forces obtained by conventional first order theory when these limits are not exceeded.

The paper studies such limits for unbraced (sway) members and for transversely loaded braced members. Existing limits are reviewed, major factors that affect lower limits are investigated and slenderness limit predictions are obtained from numerical nonlinear analyses. Finally, a new lower slenderness limit formulation is presented. The slenderness is defined in terms of a so-called normalised slenderness. In addition to the geometrical slenderness, it is a function of axial force and reinforcement. The formulation is rational and may replace, or used as an alternative to, existing lower limit formulations.

KEYWORDS

Columns (supports); Compression members; Non-slender members; Reinforced concrete; Lower slenderness limits; Second order effects; Slenderness effects; Nonlinear analysis; Sway members; Unbraced members; Braced members; Transverse loading.

Content

1 Introduction

In structures with slender compression members (columns, struts, etc.), the effect of axial forces on the displacements will affect sectional forces, and in particular the moments. These effects, generally termed second order or secondary effects, are small and can be neglected in a great many structures. In recognition of this, most design codes and standards for reinforced concrete structures give lower slenderness limits for compression members, and allow structures to be designed for forces obtained by conventional first order theory when these limits are not exceeded.

In a survey carried out in conjunction with the 1971 revision of the American Building Code ACI 318-71, it was found that the lower slenderness limits in that revision allowed second order effects to be neglected in as many as 90% of columns in braced frames, and in 40% of columns in unbraced frames (MacGregor, Breen and Pfrang 1970). These findings demonstrated the significant usefulness of the lower limits. They have found wide acceptance and have been adopted in many codes and standards internationally, in the exact same form, or in very similar forms.

For unbraced (free-to-sway) columns, which is a major concern of the present paper, the limits have in the past in general been given only in terms of effective lengths of the compression members. Use of the effective length accounts for different boundary conditions. Unlike in the case of braced members without transverse loading between ends, the effect of first order moment distributions along a member was not explicitly accounted for. Several efforts have been made to develop more refined limits that also include the effect of important parameters such as axial load (Menegotto 1983; EC2 (CEN 1991); CEB-FIP MC90 (CEB 1993); MacGregor 1993) and axial load as well as reinforcement (McDonald 1986; Hellesland 1987, 1990, 1993; EC2 (CEN 2002); Mari and Hellesland 2002). The Menegotto proposal was adopted by the Danish DS 411 (DIF 1984) and the Hellesland proposal by the Norwegian NS 3473 (NSF 1989).

A disadvantage of axial force independent limits is that they tend to be very conservative at lower axial force levels and may even be unconservative at higher levels. The degree of conservativeness or unconservativeness is also a function of the reinforcement level. Also the more refined limits give rather different results for some combinations of influencing parameters, despite inclusion of the additional parameters . This may not strengthen the confidence in their use as documentation of when second order effects may be neglected. It is important that the limits are reliable, and also not too conservative.

These observations form the motivation for this study, which is carried out along similar lines adopted in the derivation of the NS 3473 limits. They were derived (Hellesland 1987, 1990) based on elastic methodology and verified against results from a wide range of cantilever columns using nonlinear section analysis in combination with an *assumed displacement shape* (one point collocation).

The main objective of the present report is to present analysis results that accounts both for nonlinear section and geometric nonlinear (multi-point collocation) member response that allow a more rigorous evaluation of a lower slenderness limit formulation in terms of axial force and reinforcement. The scope is limited to individual compression members that are either 1) unbraced (not braced against sidesway), or that are 2) braced and have transverse loads between supports. In this pursuit, some existing limits will be reviewed and some major factors that affect lower slenderness limit predictions investigated. The factors considered include

- criterion for lower slenderness limit;
- alternative slenderness parameter (normalised slenderness);
- axial force and reinforcement (amount and arrangement);
- first order moment distribution along the member;
- sustained loading.

Several of these factors have not previously been considered in any depth, and are discussed and clarified. Results of the rather extensive nonlinear analyses presented allow assessment of existing slenderness limits and provide a basis for development of more rational limits that may be used with increased confidence. One such formulation is proposed. It is given in terms of a so-called normalised slenderness that facilitates a rather more rational and reliable definition of slenderness than the standard geometrical slenderness. In total, the proposed limit will allow the member slenderness effects to be ignored in many more cases than allowed by many current limits for unbraced members. It is demonstrated that the limit may be used with confidence also for braced members with transverse loads between supports.

Existing nonlinear analysis data, available in the literature, are employed to document sustained load effects. All presented results have been obtained for members with symmetrical reinforcement and uniform cross section, reinforcement and axial load along the member axis.

Members in braced frames with no transverse loads between their ends (i.e.,end loaded members) are considered in a separate report (Hellesland 2002).

2 Overview of lower slenderness limits

An overview of selected lower limits for unbraced members is given in Table 1. The slenderness ratio λ in the table is defined by

$$\lambda = \frac{L_e}{i} = \frac{\beta L}{i} \tag{1}$$

where i the radius of gyration of the cross section and $L_e = \beta L$ is the effective length (buckling length) and β is the effective length factor (buckling length factor). With the specific definitions of i given below (i_g, i_c), λ is simply a geometrical slenderness parameter.

The given limits for unbraced members are in the various codes and standards generally obtained from the case of braced, end loaded members by setting the end moment ratio equal to unity. The same limits are generally specified for braced members with transverse loads between their ends. However, this is not so in ACI that does not give lower limits for transversely loaded braced members. Neither does the Canadian A23.3 (CSA 1994), that also gives no limit for members that are not braced against sidesway.

In the expressions in the table, λ is defined with an i that is generally taken as that of the gross section, i_g, or, in a few cases, as that of the net concrete section, i_c. However, the difference between these is academic since i_c is approximately equal to i_g for practical steel percentages. The length L is generally either the unsupported length L_u (as in ACI 2002), the system length or something in between (CEB 1993). Use of the system length will be conservative.

The design compressive strength of concrete f_{cd} is defined somewhat differently in the different sources. In NS 3473, f_{cd} is identical to the peak stress in the concrete design stress-strain diagram and defined by $f_{cd} = f_{cn}/\gamma_c$, where γ_c=1.4 (1.25) for the ULS (Ultimate Limit State) and f_{cn} is the nominal structural ("in situ") strength. It is given by $f_{cn} = 0.70 f_{ck} + 2.8$ (MPa) for cylinder strengths f_{ck} [1] between 12 and 44 MPa, and $f_{cn} = 0.56 f_{ck} + 9.0$ (MPa) for f_{ck} between 44 and 94 MPa. In MC90 (CEB 1993) and Eurocode 2 - prestandard (CEN 1991), $f_{cd} = f_{ck}/\gamma_c$ with γ_c=1.5 for the ULS. The peak stress in the concrete design stress-strain diagram, here labelled f_c, is given by $f_c = 0.85 f_{cd}$ and $f_c = \alpha f_{cd}$, respectively, in the two codes. The factor α is defined as a reduction factor for sustained compression that generally may be assumed to be 0.85, i.e., the same as in MC90. The relative axial loads in Case 7 and 8 is in other words not defined with the peak concrete compressive stress. Therefore, ν in Case 7 and 8 is 0.85 times ν in NS 3473 and the proposal in EC2. The design yield strength of

[1] Note that in NS 3473, f_{cck} denotes cylinder strength and f_{ck} cube strength

Table 1: Overview of selected lower slenderness limits

No.	Source	Unbraced (sway) member	Details
1	ACI 318-71, ...-02	$\lambda = 22$	
2	DIN-1045 (1978)	$\lambda = 20$	
3	MC78 (CEB 1978)	$\lambda = 25$	
4	DS 411 (1984), Menegotto (1983)	$\lambda\sqrt{\nu} = 20$	
5	McDonald(1986)	$\lambda = \sqrt{\frac{125}{N_f/N_o}}$	$N_o = 0.85 f_{ck} A_c + f_y A_{st}$
6	NS 3473-1989, Hellesland (1987,1990)	$\lambda_N = 10$	$\lambda_N = \lambda\sqrt{\nu/(1+k_t\omega_t)}$ k_t=4 for "corner" reinforced rect. section; 8/3 otherwise
7	"EC2" (Eurocode 2), pre-stand. (CEN 1991)	$\lambda = \frac{15}{\sqrt{\nu}} \geq 25$	
8	MC90 (CEB 1993)	$\lambda = \frac{7.5}{\sqrt{\nu}} \geq 12$	
9	"EC2" (Eurocode 2), draft (CEN 2002)	$\lambda = 14\frac{AB}{\sqrt{\nu}}$	$A = 1/(1+0.2\phi_{ef})$ $B = \sqrt{1+2\omega_t}$

• The notation here may deviate from that used in the original sources
• $\nu = N_f/(f_{cd}A_c)$ • $\omega_t = (f_{yd}A_{st})/(f_{cd}A_c)$ • f_{cd} = design compressive strength of concrete, see text. • $f_{yd} = f_y/\gamma_c$ = design yield strength of steel.
• Re. no.9: ϕ_{ef}= effective creep factor; if not known, A=0.7 may be used. If ω_t is not known, B=1.1 may be used. Creep effects can be ignored if the final creep factor $\phi_\infty \leq 2$, $\lambda \leq 75$ and $e_0 \geq h$.

the reinforcing steel is defined with γ_s=1.25 (1.15) in NS 3473 and γ_s=1.15 in In MC90 and Eurocode 2. In the draft for the revised Eurocode 2 (CEN TC250/SC2 2002), the notation f_{cd} is, like in NS 3473, used for the peak stress. It is defined by $f_{cd} = \alpha_{cc} f_{ck}/\gamma_c$, where the sustained compression factor α_{cc} may be taken between 0.85 and 1.0.

Although not always explicitly stated in the various sources, the given limits have been obtained for and tacitly apply to members with approximately uniform cross-section and reinforcement along the member.

According the ACI 318-71 Commentary, "the lower limits (were) determined from a study of a wide range of columns and correspond to lengths for which a slender member strength of at least 95% of the cross-sectional strength can be developed". The strengths referred to here are in terms of moments rather than axial loads. This is apparent from the ACI 318-95 Commentary where it is stated that the limit was derived from the moment magnifier expression "assuming that 5% increase in moments due to slenderness is acceptable". It may be noted here that a specified moment increase of 5% in the moment magnifier approach (that treat all member failures, incl. instability, through material failures) is approximately identical to a moment capacity reduction to 95% of the cross-sectional moment capacity (1/1.05= 0.952).

The limit proposed by Menegotto was related to 10% moment increase or 10% increase in required reinforcement. It was based on a nonlinear study of eccentrically loaded cantilever columns (i.e., with uniform first order moment distribution). This limit was adopted in the Danish standard DS 411 for compression members in general.

McDonald derived his lower limit from unrestrained (hinged) columns, taking a 5% increase in first order moments as being acceptable.

In the derivation of the NS 3473 limit, the criterion was 5% reduction in moment capacity for practical cases (Hellesland 1990). The slenderness parameter λ_N is termed "axial force dependent slenderness" in NS 3473.

In the prestandard version of Eurocode 2, the limit was related to 10% increase in moments above the first order moments. The CEB-FIP Model Code MC90 states that lower slenderness limits should be related to the reduction in bearing capacity of not more than 10%. According to MC90, the given limit is derived for minimum reinforcement.

To what extent, if any, creep due to sustained loading has been included in the derivation of the various limits, is not always clear from the presentations. In general, it is believed that creep effects either have been considered negligible at the lower limits or that the given limits were considered sufficiently conservative to tacitly allow for a certain amount of creep.

Regarding the ACI 318 code, neither the original paper (MacGregor et al. 1970) nor the commentaries give any details pertaining to creep effects in the limits. This also applies to McDonald's proposal.

Based on analyses with a final creep factor of ϕ_∞=3.5, Menegotto (1983) found that creep effects on his eccentrically loaded columns could be ignored up to slenderness values beyond his proposed lower limit. Indeed, he proposed that creep need not be considered for slendernesses less than $\lambda\sqrt{\nu_s}$=20, where ν_s is the sustained service axial load level. For a sustained load ratio of ν_s/ν=0.6, this implies a slenderness that is 29% larger than the lower limit (Table 1). A similar creep provision (replacement in the lower limit formulation of the ultimate load (ν) by the sustained load (ν_s)) was adopted by the Norwegian NS 3473. In MC90, no comments are made as to a possible inclusion of creep effects.

The prestandard version of EC2 (CEN 1991) has been in the process of being revised for some time. Several slenderness limits have been proposed at various stages in the revision process (CEN 1999; Westerberg 1999; CEN TC250/SC2 2001; Westerberg 2002; CEN TC250/SC2 2002), all related to a 10% capacity reduction. During the process, the author has commented and presented alternative proposal and suggestions for changes (Hellesland 1999, 2001, 2002a). The slenderness limits in the "final" draft by CEN TC250/SC2 (2002), given by Case 9 in Table 1, is reasonably similar to that of NS 3473 except that it is formulated in terms of the geometrical slenderness parameter. A major differences is the inclusion of creep in the proposed EC2 limit.

3 Criterion

In elastic analyses, a slenderness criterion may appropriately be related to a specified moment increase as it is mainly the moment computation that is affected by the neglect of second order effects. In reinforced concrete structures, with nonlinear material properties, an objective lower slenderness criterion can most appropriately be related to an acceptable percentage reduction in load capacity rather than to some moment increase.

A general lower slenderness limit is here defined as that at which second order load effects (slenderness effects) do not reduce a member's load carrying capacity to values less than a specified percentage (95 or 90%) of the cross-sectional capacity ("non-slender (short) member strength"). Non-negligible detrimental effects of sustained (long term) loading must, if relevant, be included in capacity reduction assessments.

Within such a general definition, it is possible to envisage several more detailed criteria on which to base calculations leading to an approximate limit expression. These include

a) 5% (10%) reduction (ΔM) in moment capacity (M_d) for a constant axial load;
b) 5% (10%) reduction (ΔN)in axial load capacity (N_d) for an applied constant axial load eccentricity (implies also a 5% (10%) reduction in moment capacity);
c) 5% (10%) reduction (ΔN) in axial load capacity (N_d) for an applied constant moment.

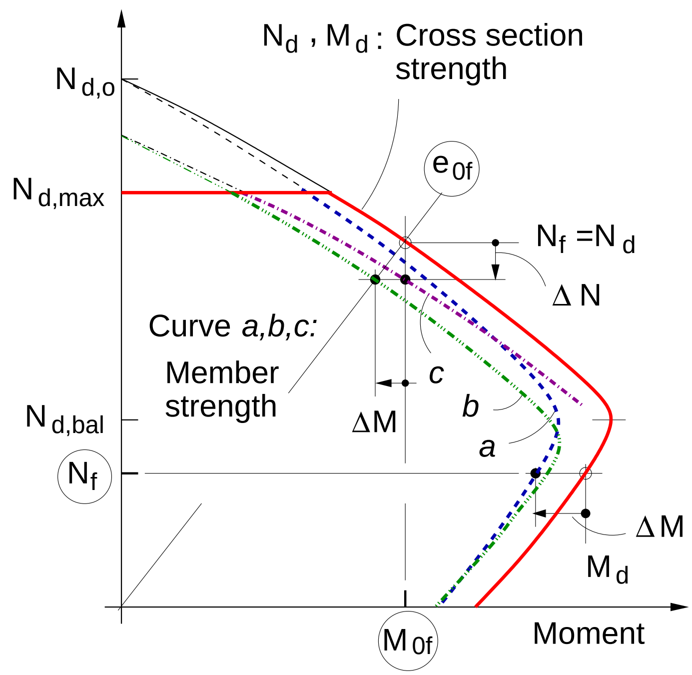


Figure 1: Capacity reductions according to various slenderness limit criteria

The remaining capacity for external loads, i.e., the so-called first order capacity or (slender) member strength, is defined schematically in the $N-M$ diagram in Fig. 1 for the the three cases defined above. The different capacity reductions are reflected by the difference between these and the curve for the cross section strength (non-slender member strength). N_f and M_{0f} are the factored axial load and the maximum factored first order moment along the member, and e_{0f} =

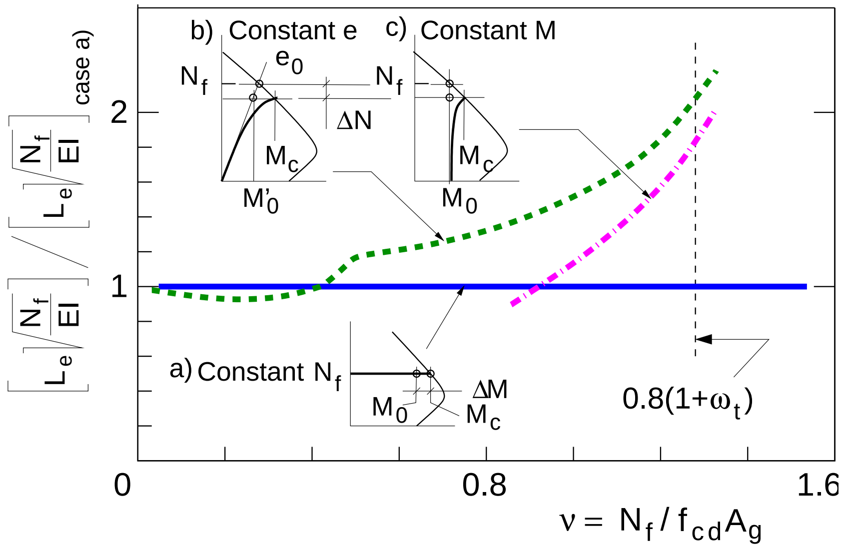


Figure 2: Slenderness limits according to various definitions (based on moment multiplier) for a medium reinforced member (5% red., $\omega_t = 0.6$)

M_{0f}/N_f the corresponding axial load eccentricity. For simplicity, the subscript f, will be deleted in the remainder of the report in the first order moment and eccentricity notation (M_0, e_0). N_d and M_d are the design axial load and moment capacity of the cross section.

For the purpose of illustration, approximate values corresponding to the three cases defined in Fig. 1 are shown in Fig. 2. They have been calculated with data from a N-M diagram for a typical medium reinforced cross section ($\omega_t = 0.6, h'/h = 0.8, \varepsilon_{yd} = 0.0024$, stress-strain diagrams as defined in Fig. 6) in conjunction with the approximate moment magnifier approach (e.g., ACI 2002). With the moment gradient factor C_m=1, this approach gives

$$\frac{kL}{\pi}\sqrt{\frac{N}{EI_d}} = \sqrt{1 - \frac{1}{\delta}} \tag{2}$$

where N is the axial load (defined positive in compression) and EI_d is the design section stiffness. Since also primary instability is considered through material (section) failure in the moment magnifier approach, the design maximum moment, defined by $M_c = \delta M_0$, is equal to the nominal moment capacity ($M_c = M_d$) at full section utilisation.

All results (Fig. 2) are expressed relative to the results for Case a. The slenderness limit in terms of $kL(N_f/EI_d)^{1/2}$ is for this case independent of section details (N-M diagram) and becomes equal to a constant value (= 0.218 for $N = N_f$ and $\delta = M_c/M_0 = 1/0.95$ =1.053) according to Eq. 2. This is not so for the other two cases, for which, on the overall, the limit is seen to increase with increasing axial load level. For these cases, the moment magnifier $\delta = M_c/M_0$ is based on moment values read from the chosen $N - M$ interaction diagram, as indicated by the inserts in Fig.2. Note that N in Eq. 2 must be taken as $0.95N_f$ for these cases (b and c).

It can be seen (Fig. 2) that criterion a gives more conservative limits than criterion b except at lower relative axial load levels (less than approximately the balanced load at 0.43) for which the difference in any case is rather small. At higher axial loads, the difference can become quite substantial. Criterion c gives results between the two others. The lower terminations of the Case c curve corresponds to an applied moment (M_0) approximately equal to the pure moment capacity. Case c, as defined, is therefore not relevant for larger moments.

Criterion a may be the most appropriate for a more lowly axially loaded member with larger load eccentricities, for which the moment capacity may be the most important strength parameter. For such members, effects of creep are small and often insignificant. Beam-columns and, in particular, unbraced members, are in this category. Criterion a may be unduly conservative at high axial loads if creep effects are also to be included.

For highly compressed members with smaller load eccentricities, for which the axial load capacity may be the most relevant strength parameter, a criterion such as b or c may be considered the most appropriate. Columns in multistory structures, and in particular those in lower stories, are typically in this category. It is for such members that possible effects of creep are strongest. Creep effects will lower the curves, increasingly so at high load levels. If creep effects were to be included in criterion b and c, but not in criterion a, the curves for b and c will approach the one for a.

A general purpose slenderness limit criterion must be chosen reasonably conservative, yet reflect to a reasonable extent realistic loading situations that may be encountered in practice. Also, it is advantageous that it is reasonably simple to incorporate in practical computations.

These premises are found to be reasonably fulfilled by the sole use of criterion a with short term material properties, and is the approach adopted here. At higher axial loads it is, as will be demonstrated, sufficiently conservative relative to criterion c to indirectly allow for a reasonably amount of creep in c. Criterion b would even allow more creep, but may be too liberal for the general case.

4 Normalised slenderness

The sensitivity to second order bending effects of elastic compression member with negligible shear effects is a function of the combined parameter $L(N/EI)^{1/2}$ rather than of the individual parameters entering the expression (L, N and EI). In addition it is a function of boundary conditions (end restraints). In the trigonometric solution of the differential equation for the member, $L(N/EI)^{1/2}$ represents the member length. Such "length" parameters, "normalised" with respect to the effect of N and EI, and with L replaced by the effective lengths (L_e) to account in an approximate manner for different boundary conditions, would seem like a rational choice in slenderness limit formulations also of reinforced concrete members.

In order to adapt to reinforced concrete members, a representative sectional bending stiffness may be given by the secant stiffness defined by

$$EI_d = k_c E_{cd} I_g + k_s E_{sd} I_s \tag{3}$$

Within the partial safety factor philosophy, the EI_d above is a design value to be used in conjunction with the required axial load capacity $N_f = N_d$. Here I_g and I_s are the second area moments of the gross section (for simplicity, rather than I_c of the net concrete section) and the total reinforcing steel about the centroidal axis, respectively, k_c and k_s are coefficients, E_{cd} is the design elastic modulus of the concrete and E_{sd} of the steel. Some codes take $E_{sd} = E_s$ (MC90, CEN 1991, etc.), i.e. without reducing it by a safety factor. On substitution of EI_d, the combined slenderness parameter may be expressed by

$$L_e \sqrt{N_f / EI_d} = \lambda_n / \sqrt{f_{cd} / k_c E_{cd}} \tag{4}$$

where

$$\lambda_n = \lambda \sqrt{\frac{\nu}{1 + k_t \omega_t}} \tag{5}$$

in which

$$\lambda = \frac{L_e}{i_g} \quad ; \quad \nu = \frac{N_f}{f_{cd} A_g} \quad ; \quad \omega_t = \frac{f_{yd} A_{st}}{f_{cd} A_g} \tag{6}$$

is the geometrical slenderness, the non-dimensional factored design axial load level and the total mechanical reinforcement ratio, and k_t is a constant for given cross-section, reinforcement and axial load. Further, f_{yd} is the design steel yield strength and f_{cd} is the design concrete compressive strength defined as the peak stress in the design stress–strain diagram for concrete.

NS 3473 and other codes give a structural strength to cylinder strength ratio, f_{cn}/f_{ck}, that decreases with increasing strength. This and alternative variations are discussed in the literature (Collins et al. 1993, Ibrahim and MacGregor 1997, etc.). Similarly, f_{cd}/f_{ck} will vary with increasing strength. Smaller changes in this ratio does not affect λ_n too much since they have counteracting effects in the numerator and denominator of Eq. 5. Therefore, if desired, a simplified constant relationship between f_{cd} and f_{ck} may be adopted.

The slenderness parameter, λ_n, expressed completely in terms of nondimensional quantities, is well suited for reinforced concrete members, and can appropriately be labelled **normalised slenderness** (Hellesland 1993) to distinguish it from other slenderness parameters. Such a parameter has been used in NS 3473 since 1989, and there denoted λ_N and labelled "axial load dependent slenderness".

Finally, consistent with the assumed secant stiffness expression,

$$k_t = \frac{(k_s E_{sd}/f_{yd})}{(k_c E_{cd}/f_{cd})} \left(\frac{i_s}{i_g}\right)^2 \tag{7}$$

where $i_g = (I_g/A_g)^{1/2}$ and $i_s = (I_s/A_{st})^{1/2}$ are the radii of gyration of the gross section and of the total longitudinal reinforcement, respectively, both about the centroidal axis.

The k_c and k_s factors are functions of axial load level and of other factors, but simplifications are warranted, in particular with respect to k_s, that may be taken equal to unity, and the quantity $k_c E_{cd}/f_{cd}$, that may be taken equal to a constant. A simplified k_t–expression in the form of

$$k_t = \frac{4.3}{1000\,\varepsilon_{yd}} \left(\frac{i_s}{i_g}\right)^2 \tag{8}$$

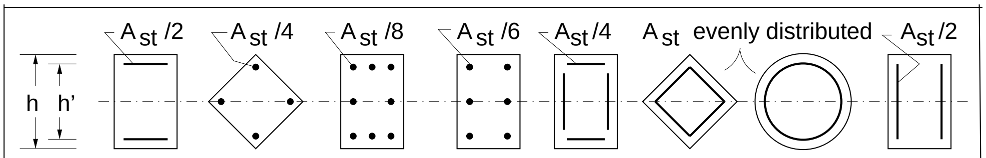
where $\varepsilon_{yd} = f_{yd}/E_{sd}$, is adopted in this study. To comply with the adopted safety philosophy of codes that do not reduce the modulus of the steel, E_{sd} above must be taken equal to E_s. The validity of this expression will be evaluated and discussed below.[2]

Whereas k_t, as defined above, is dependent on steel quality through ε_y, the product $k_t\omega_t$ is not. It can be given by

$$k_t\omega_t = \frac{4.3 E_{sd} A_{st}}{1000 f_{cd} A_g} \left(\frac{i_s}{i_g}\right)^2 \tag{9}$$

It is a question of preference whether to use the latter form or to use Eq. 8 in combination with ω_t.

[2]With the common values of k_s=1 and k_c=0.2, Eq. 7 gives a 35% greater k_t–value than above when $E_{cd} = 870 f_{cd}$. The latter is the secant modulus at σ/f_{cd}=0.45 of the parabola-rectangle stress-strain curve adopted in the nonlinear analyses in this study (with $\varepsilon_{co} = 0.002$).



i_g/h	0.289	0.204	0.289	0.289		0.204	0.250	0.289
i_s/h'	0.500	0.354	0.433	0.408		0.289	0.354	0.289
i_s/i_g	1.732 h'/ h		1.500 h'/ h	1.414 h'/ h				1.000 h'/ h
k_t 1)	4.0		3.0	4.0 x 2/3 = 2.66				1.33
k_t 2)	3.2		2.4	3.2 x 2/3 = 2.13				1.07

1) ε_{yd} = 0.002, h'/ h = 0.79 2) ε_{yd} = 0.0025, h'/ h = 0.79

Figure 3: Typical radii of gyration of gross cross-sections and of symmetrical reinforcing bar arrangements.

Values of i_g, i_s and i_s/i_g for typical cross-sections and reinforcement arrangements are given in Fig. 3. Typical k_t–values defined by Eq. 8 for two steel qualities are also given.

There is room for simplifications to be made in Eq. 8 provided some conservativeness is accepted. If it should be considered desirable with a section independent value, a steel quality independent value, or simply a constant value, it might be considered sufficiently conservative to adopt either

$$k_t = \frac{6}{1000\,\varepsilon_{yd}} \quad ; \quad k_t = 1.8\left(\frac{i_s}{i_g}\right)^2 \quad or \quad k_t = 2.5 \tag{10}$$

respectively. In these examples, either i_s/i_g=1.48h'/h with h'/h=0.8, ε_{yd}=0.0024, or both of these have been introduced, respectively. Alternative simplifications can be obtained with other values of these parameters.

5 Nonlinear analyses

Method – Specified moment capacity reduction

The member analysed is a laterally unbraced cantilever column, clamped at the base an free at the upper end. In accordance with the normal practice for reinforced concrete members, it was assumed that the members were initially straight prior to loading. Further, the members considered had uniform axial load and uniform section details and material properties along the length.

The member and the various external loads considered in this study, and corresponding first order moment distributions, are defined in Fig. 4(a1 to a4).

Clearly, also the case in Fig. 4(b) can be modelled by a cantilever column (case a3). Further, it may be used to model a series of braced member cases with either end moments or transverse loads between ends. All but the distributed load case (c) in Fig. 5 can be modelled by a cantilever column. For all cases shown, maximum moments due to second order effects and maximum first order moments result at the same section.

A finite difference approach was adopted for the computation of slenderness values causing a specified reduction in first order moment capacities of the reinforced compression members. It was tailor made for the problem and include both nonlinear material and nonlinear geometric effects (Aasrum 1992). An overview of the major steps in the iterative analysis is given in Appendix B. A member may become unstable either due to primary material failure (exhaustion of the cross-section capacity) or primary instability failure prior to material failure. The basis

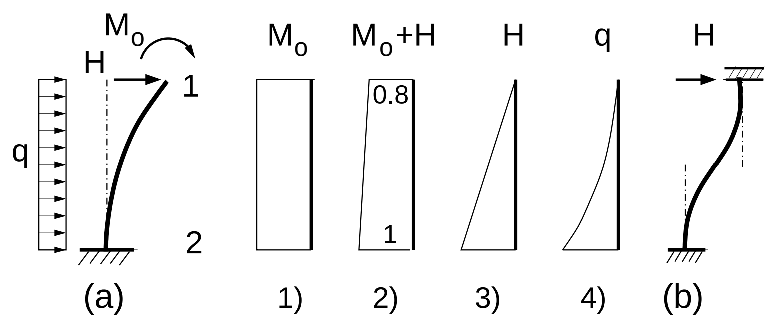


Figure 4: Unbraced (sway) column with end and transverse loading – First order moment distributions

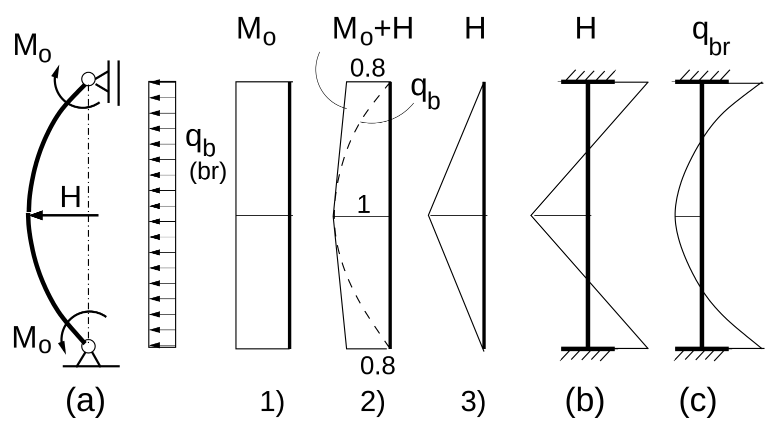


Figure 5: Braced column with end and transverse loading – First order moment distributions

for the analysis was the relationship between the curvature ($\kappa = 1/R$) and the nominal moment resistance at that curvature ($M_{d,\kappa}$). Such $M_{d,\kappa} - \kappa$ relationships for a given section, reinforcement and nominal axial load were computed from the equilibrium equations for axial load and moment (about the centroidal axis);

$$N_f = N_d = \int_{A_c} \sigma_c \, dA_c + \int_{A_{st}} \sigma_s \, dA_s \tag{11}$$

$$M_{d,\kappa} = \int_{A_c} \sigma_c z \, dA_c + \int_{A_{st}} \sigma_s z \, dA_s \tag{12}$$

The standard assumptions of plane sections remaining plane (Bernoulli-Navier's hypothesis), full bond, neglect of the concrete tensile strength and the favourable tension stiffening (giving increased stiffness between cracks), were incorporated. The commonly adopted parabola-rectangle diagram, Fig. 6(left), was chosen for concrete in compression and a standard elasto-plastic stress-strain diagram, Fig. 6(right), for reinforcing steel in tension and compression. The effect of unloading

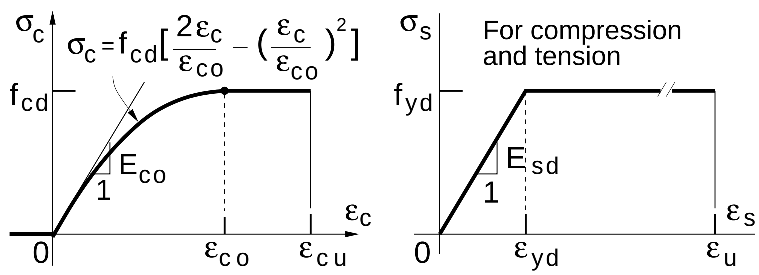


Figure 6: Stress-strain diagrams for concrete (left) and reinforcing steel (right)

that may take place in some fibres is small and was not accounted for. Non-mechanical concrete strains (creep, shrinkage) were not included. Consequently, results obtained are so-called "short term" results.

The material factors γ_m (γ_c, γ_s) were assumed to be the same at all sections. This is a common, but conservative approach. At sections outside the the most strained (critical) region, where failure is initiated, use of smaller factors would have been justified to better represent the stiffness of these regions. Compared to such a double-factor approach, use of the same, higher factors for all sections will underestimate member stiffness, give larger displacement and somewhat too small slenderness values.

Parameters

Slenderness values at which the first order moment capacity (i.e., the member's capacity for carrying first order moments caused by external loads) is reduced by 5 and 10% below the the cross-sectional moment capacity ($M_d = M_{d,\kappa}$ at the ultimate (maximum) curvature) have been computed for the initially straight, uniform members for a wide range of parameters. These include rectangular and circular section shapes, different symmetrical reinforcement arrangements ("corner", distributed etc.), reinforcement locations ($h'/h = 0.7, 0.8, 0.9$), steel yield strains ($\varepsilon_{yd} = 0.002, 0.0025, 0.003$) and ultimate concrete compressive strains (ε_{cu}=0.003, 0.0035). Unless otherwise mentioned specifically where relevant, the

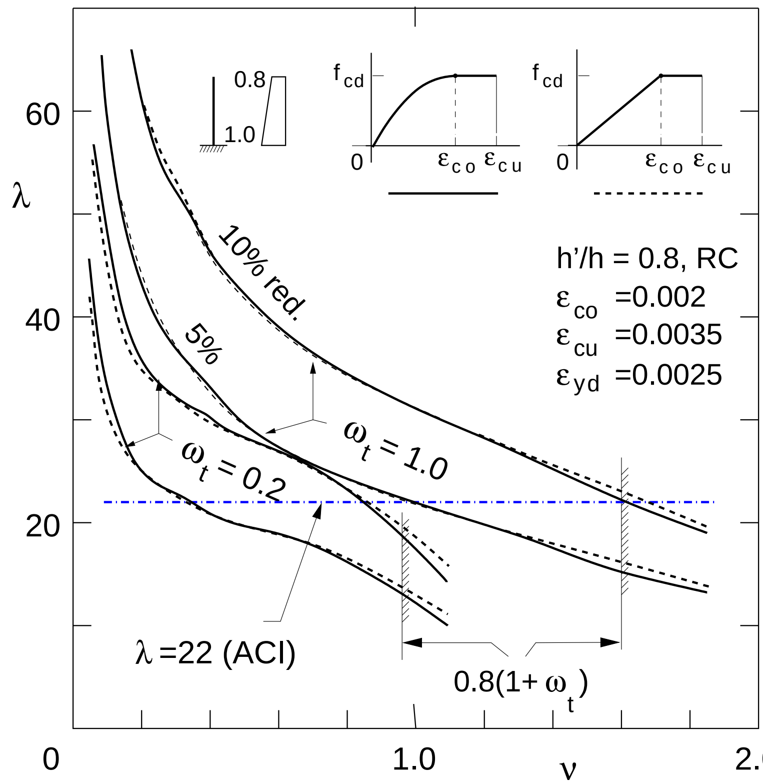


Figure 7: Slenderness vs. axial load level for 5 and 10% capacity reduction, two reinforcement ratios and two concrete stress–strain diagrams

results presented are for members with rectangular cross-sections and reinforcement in two opposite layers perpendicular to the plane of bending, or equivalent (e.g., corner reinforcement). This section is labelled RC. Further, $h'/h = 0.8$, $\varepsilon_{yd} = 0.0025$, $\varepsilon_u = 0.010$, $\varepsilon_{co} = 0.002$ and ε_{cu}=0.0035.

In results involving λ_n, k_t is taken according to Eq. 8. For h'/h=0.8 and ε_{yd}=0.0025, it becomes k_t=3.3 for an RC–section. For a for a rectangular section with distributed, equal reinforcement in each face, labelled RD, $k_t = 2.2$ (2/3 times 3.3).

Considered mechanical reinforcement ratios are in the range $\omega_t = 0.1 - 1.2$, with most results obtained for $\omega_t = 0.2$ and $\omega_t = 1.0$. These cover a wide range from approximately minimum reinforcement to an, in practice, upper limit.

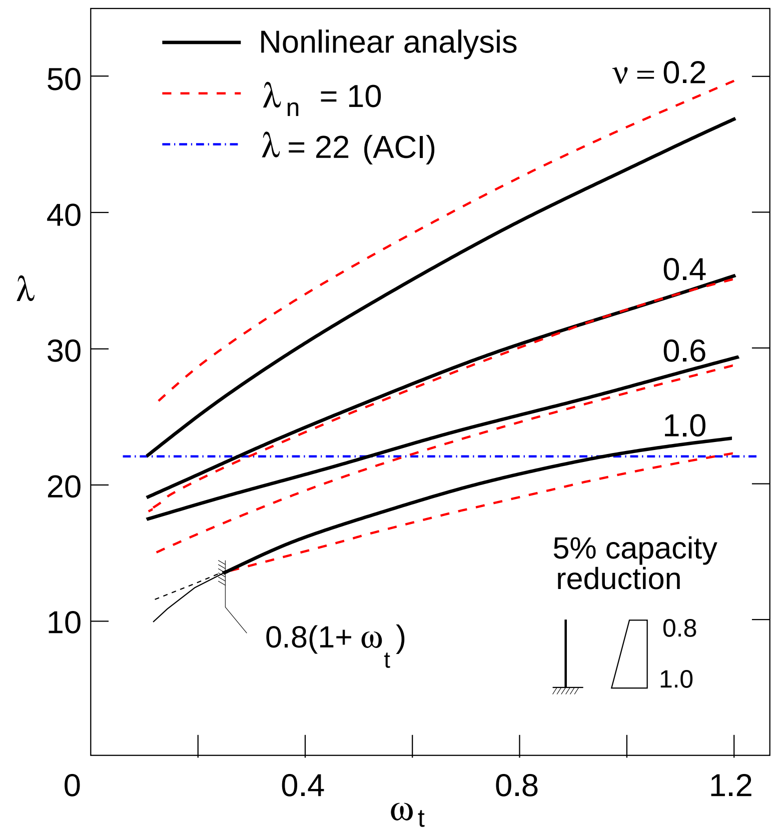


Figure 8: Slenderness vs. reinforcement for different axial load levels.

Axial force and reinforcement

Two major factors affecting the slenderness limit are axial load and reinforcement. Their effects on the geometrical slenderness ($\lambda = L_e/i_g$) limits are shown in Fig. 7 and Fig. 8. The axial force is the more important of these factors, as is obvious from Fig. 7, but it can be seen that reinforcement effects also are quite substantial.

These results are for a slightly non-uniform first order moment distribution with a top moment equal to 0.8 times the bottom moment (cf Fig. 4(a1), Fig. 5(a1)). For convenience it will subsequently be labelled “slightly non-uniform”. Similar $\lambda - \nu$ variations result for other first order moment distributions. For a uniform distribution, the curves would have had more marked slope discontinuities near the balanced load.

The axial load of 80% of the pure axial load capacity (“squash load”) is shown by the vertical lines in this and other figures. This represents an approximate maximum upper load in many codes, and corresponds to a relative end eccentricity of the axial load of about e_0/h=0.1.

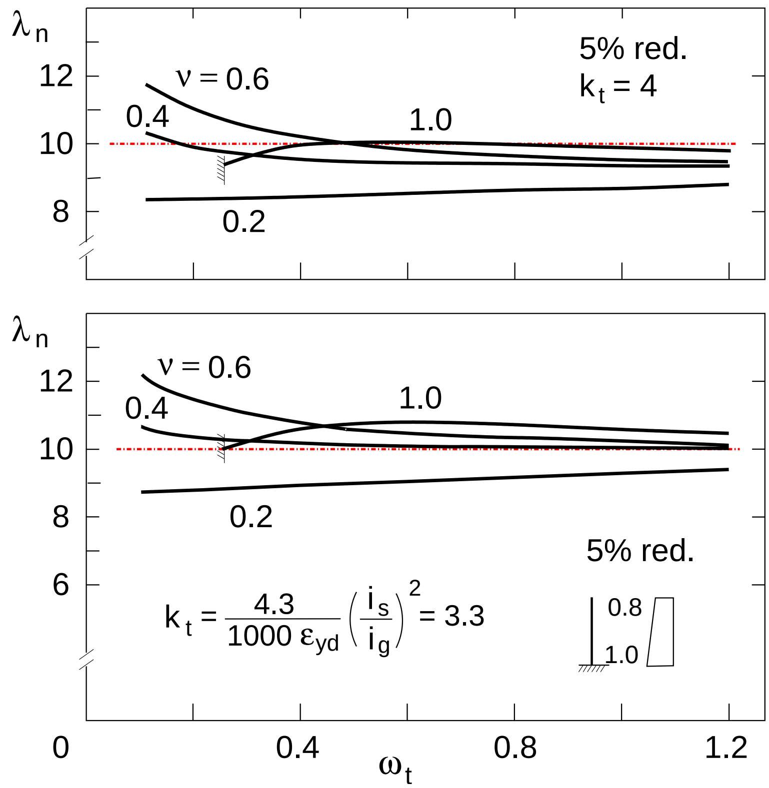


Figure 9: Normalised slenderness vs. reinforcement for different axial load levels and k_t=3.3 (bottom) and k_t=4 (top)

Constant in the k_t–expression

In order to evaluate the proposed reinforcement contribution factor k_t (Eq. 8), a normalised slenderness λ_n=10 is also shown in Fig. 8 (k_t=3.3). It is seen to be in reasonably good agreement with the nonlinear results. In particular for the axial load curve of ν=0.4 (close to the balanced load). At this load it can be seen that the correspondence is about equally good at low and high reinforcement levels. At the other loads this is not the case to the same degree, but still good.

The discrepancies depend also on the chosen limit (λ_n=10) in the comparison, which will be discussed more later. To further assess k_t, the results in Fig. 8 are replotted in terms of λ_n in Fig. 9. An ideal k_t–factor should render all curves into one horizontal line. As this is not the case, the real k_t is clearly a function of the reinforcement level and the axial force. The largest variation results for lightly reinforced members (ω_t=0.1–0.25). In order to obtain approximate horizontal lines for each load level, it would require k_t–values of about 4.4–4.5 for ν=0.2, 3 for ν=0.4, 2 for ν=0.6 and 3.5–4 for ν=1.0. Similar results are obtained for the 10% capacity case.

The adopted axial load independent value of k_t (=3.3 in the considered case), represents a reasonable compromise. It is somewhat too small for loads above the balanced load and too small for loads below. The same value is acceptable for the 10% capacity case.

The same results are replotted in Fig. 9, top, with k_t=4 rather than with k_t=3.3. In terms of Eq. 8, this corresponds to the replacement of the constant 4.3 by 5.2. Use of the greater k_t alters the slopes of the curves (versus ω) somewhat. However, they still seem acceptable. In addition it lowers the curves. In previous studies (Hellesland 1987, 1990, 1993; Aasrum 1992), the higher constant in k_t was adopted.

Normalised slenderness presentation

The variation versus axial load and reinforcement is reasonably accounted for by the normalised slenderness definition. This is best seen in Fig. 10 which is a replot in terms of the normalised slenderness of Fig. 7. To completely "linearise" the results into horizontal lines, k_t would have to be both axial load and reinforcement dependent. The chosen constant value is seen to be reasonably good in bringing the curves together for the low and high reinforcement level.

The low reinforcement level results vary considerably with varying axial load. Results for intermediate reinforcement levels (ω_t greater than about 0.4) will be very similar to those for the high reinforcement level, which is seen in the figure to stay reasonably constant with varying axial load. The set of upper curves, giving slenderness limits for 10% capacity reduction, are approximately 35 to 45%

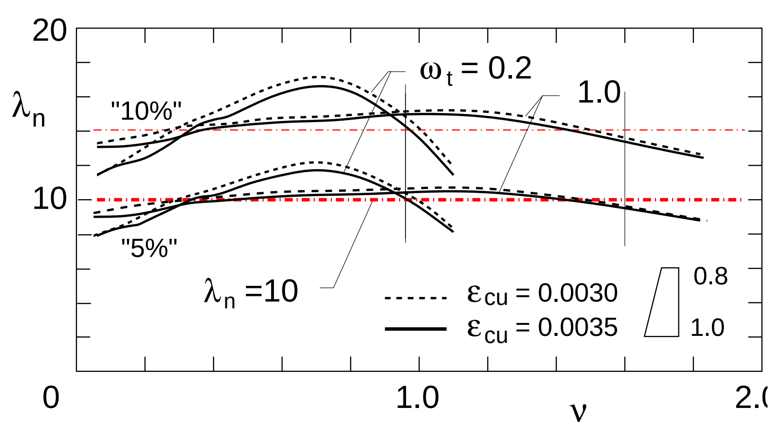


Figure 10: Normalised slenderness vs. axial load level – Effect of ultimate concrete strain (*RC* section)

greater than the set of lower curves for the 5% reduction. This is a considerable difference. It compares well with the 38% increase that the magnifier (Eq.2) gives.

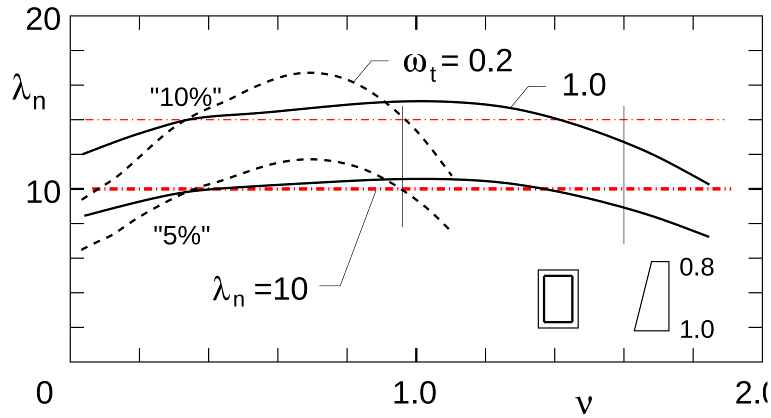


Figure 11: Normalised slenderness vs. axial load level – Rectangular section with distributed reinforcement (RD)

Fig. 11 shows corresponding results for a rectangular section with distributed, equal reinforcement in each face (RD). Results for circular sections with distributed reinforcement are very similar to these.

Applicability to different concrete qualities/practices

In the analyses, the strain at the apex of the ascending concrete stress–strain branch, ε_{co}, was kept constant at common value of 0.002. This is warranted as this strain is not much affected by the concrete strength and density. The practice regarding the ultimate concrete strain, ε_{cu}, is more varied. In the ACI 318 code, ε_{cu} is taken as 0.003. In most European codes it is taken as 0.0035. In the Norwegian NS 3473, it varies, depending on concrete quality, approximately between these values.

No dependence on ε_{cu} has been incorporated into the slenderness formulation. Based on results given in Fig. 10, for ε_{cu}=0.003 and 0.0035, this seems justified. The difference in results for these two ultimate strain values is rather small, and smaller than found in an earlier study based on an assumed displacement shape (Hellesland 1993). Limits based on ε_{cu}= 0.0035 will be conservative (to the safe side) for ε_{cu}=0.003 (at most about 5% for low reinforcement levels and less at higher levels).

The slenderness formulation is also given independent of the concrete modulus–strength ratio. In various codes (ACI 318-02, MC90, NS 3473, etc.), this ratio varies considerably with concrete quality both for normal density and light weight aggregate (LWA) concretes. In NS 3473, E_c/f_{ck} (where E_c is the unloading modulus) decreases from approximately 1170 to 430 when f_{ck} increases from 20 MPa to 84 MPa for normal density concrete. For normal density concrete according to ACI 318-02, E_c/f_{ck} (where E_c is the secant at $0.45f_{ck}$) decreases from approximately 1040 to 570 when f_{ck} increases from about 21 MPa to 69 MPa. For f_{ck}=30 MPa it is approximately 870. Also Eq. 7 (for k_t) and Eq. 4 seem to indicate a dependence on the E_{cd}/f_{cd}–ratio. It may be questioned therefore whether the presented results, obtained using only one single concrete stress–strain diagram, have limited applicability.

This question is addressed in Fig. 7, that shows slenderness limits obtained with two different concrete stress-strain diagrams: a) a trapezoidal (bilinear, elasto-plastic) diagram with $E_{cd}=500f_{cd}$ and b) the standard parabola-rectangle diagram (with a secant stiffness of $E_{cd}=870f_{cd}$ at $0.45f_{cd}$ and $E_{cd}=816f_{cd}$ at $0.6f_{cd}$). The effect of the different diagrams, with widely different E_{cd}/f_{cd}–ratios, are seen to be surprisingly small, both at the 5 and 10% capacity reduction level.

The results are dependent on the moment curvature resistance curve at the critical section and the second order moments due to displacements. For the almost uniform moment distributions, as in the considered case, displacements are influenced mainly by the upper portion of moment resistance curve. That portion is not too much affected by the difference in the two stress-strain diagrams considered. With increasingly nonlinear moment distribution, different stress-strain diagrams can be expected to affect results to an increasing extent.

The present results confirm results obtained in an earlier study in which an assumed (approximately sinusoidal) displacement shape was adopted in combination with several stress-strain diagrams (Hellesland 1993).

Reinforcement arrangement, cover, section shape, steel quality

Fig. 12 shows results for heavily reinforced rectangular sections with 3 different reinforcement arrangements labelled RC, RD and RP. The two former sections are defined before. The latter, identifying a rectangular section with distributed, equal reinforcement in the two faces parallel to the plane of bending, is rather extreme, and in practice unlikely. It is still interesting for the purpose of comparison.

The relative effect of neglecting the influence of the reinforcement arrangement can be seen in the upper part of the figure, where the results are plotted using the same value of k_t=3.3 for all 3 cases. As expected, considering that i_s in case RD and RP are smaller than that of case RC (cf. Fig. 3), the results for the two latter cases fall below those for case RC in such a plot. In the lower part of the figure, the results are plotted using k_t according to Eq. 8 (3.3, 2.2 and 1.1, respectively). Considering that the 3 curves are rather close together, and that the only variable

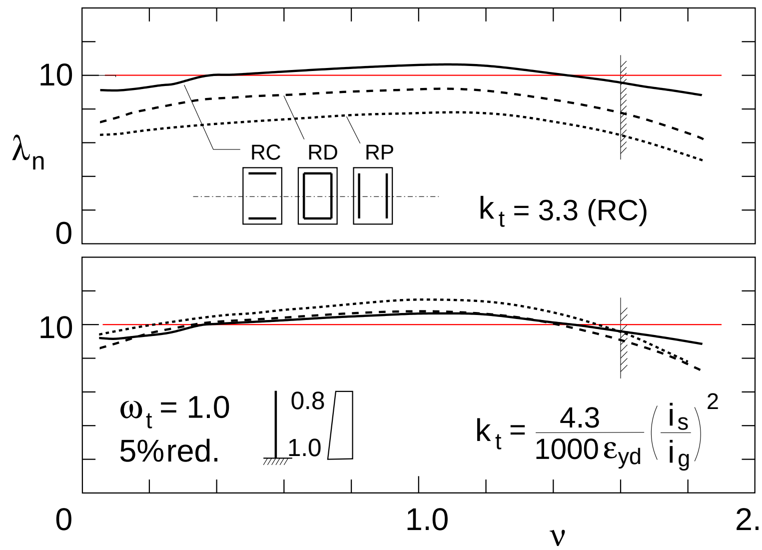


Figure 12: Normalised slenderness vs. axial load level – Effect of reinforcement arrangement (RC sections with $h'/h = 0.8$)

in the expression is i_s, it can be concluded that the use of the elastic i_s adequately reflects different reinforcement arrangements. Similar results are shown in Fig. 13 for RC–sections with different concrete covers (reinforcement locations). The relative effect of neglecting the influence of the cover can be seen in the upper part of the figure. In the lower part of the figure, results are plotted using k_t according to Eq. 8 (giving 4.2, 3.3, and 2.5, respectively, for h'/h=0.9, 0.8 and 0.7). It can be seen that the elastic i_s again reflects different covers extremely well.

From similar comparisons, Eq. 8 has been found also to reflect the effect of different yield strains (inverse proportionality) and section shapes (through the elastic i_g of the gross cross section) very well. It may be concluded that the k_t–expression reflects different cross sections, reinforcement details and steel qualities very well through the parameters i_s, i_g and ε_{yd}.

First order moment distributions

All results presented sofar are for members with a slightly non-uniform first order moment distributions ("$M_0 + H$"). With increasingly non-uniform first order moment distribution, the slenderness values will increase. This is seen in Fig. 14 and 15 for RC–sections with a low and a high reinforcement ratio, respectively.

The first order moment distributions from Fig. 4 are indicated in the figures. However, the results are applicable also to braced members. Results for cases in Fig. 4 and Fig. 5 with the same label are identical (e.g., cases (a3) and (b)

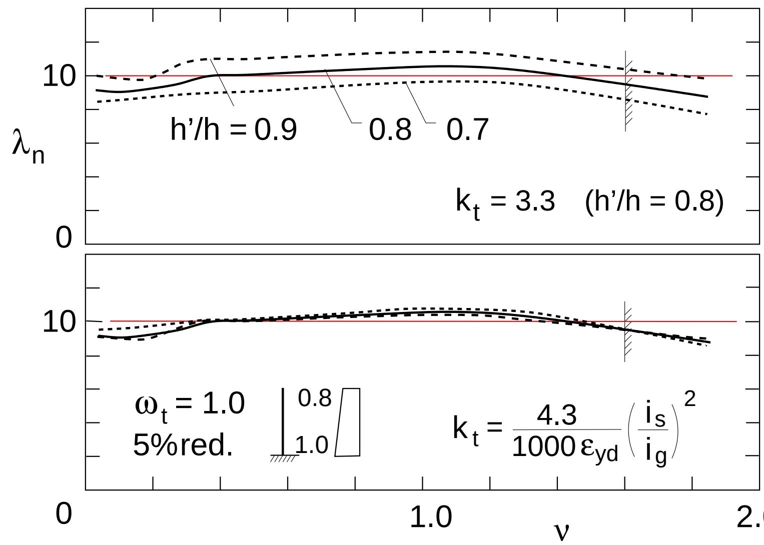


Figure 13: Normalised slenderness vs. axial load level – Effect of concrete cover (RC section)

labelled "H" in the two figures).

The distribution labelled "q", due to a constant distributed (wind) load on the cantilever (Fig. 4(a4)), deviates most from the uniform distribution and give rise to the highest slenderness values. Next to this follows the results for the triangular distributions, "H", and then those for the slightly non-uniform distributions, "$M_0 + H$".

The lowest values are obtained for the member with the uniform distribution (due to end moment and vanishing transverse loading). The marked change in slope at about the balanced load is typical for uniform moment distributions in combination with RC–sections . For other sections (RD, circular, etc.) there will be a more gradual change. As far as transversely loaded members are concerned, this is a rather academic case. It is still included as it represents a limiting case.

Results for distributed loading on braced members with hinged ends ("q_b", Fig. 5(a2, dashed line)) are not shown, but it can be inferred that they would be located between those for the slightly non-uniform ("$M_0 + H$") and the triangular ("H") distribution cases, i.e., between the two middle curves in Fig. 14 and 15. For distributed loading on braced members with restrained (clamped) ends ("q_{br}", Fig. 5(c)), results are expected to become located between the two upper curves in the figures.

Comparison with selected code limits

It is of special interest to compare results with an axial load independent limit.

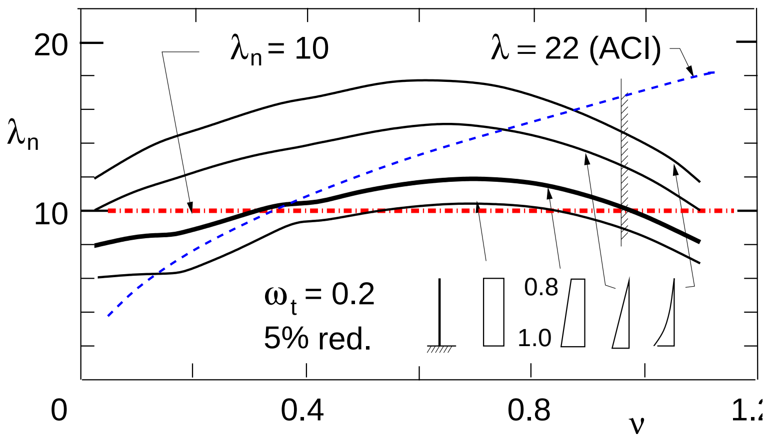


Figure 14: Normalised slenderness vs. axial load level – Effect of different moment distributions for lightly reinforced member (*RC* section)

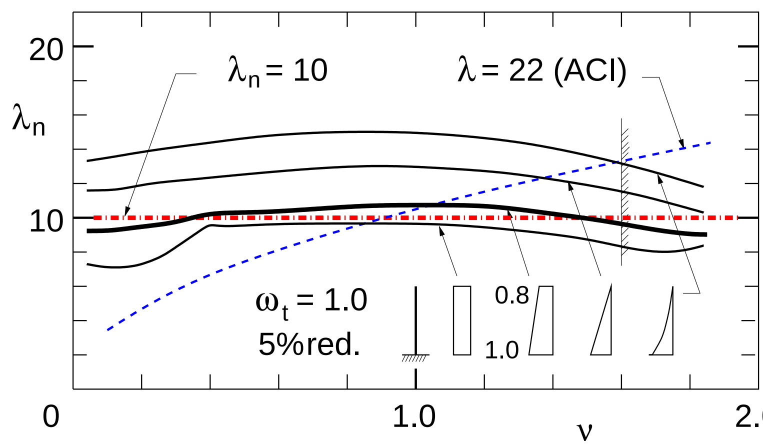


Figure 15: Normalised slenderness vs. axial load level – Effect of different moment distributions for strongly reinforced member (*RC* section)

The one given by ACI 318 is chosen for this purpose. Compared to the 5% capacity reduction results for the "slightly non-uniform" first order moment case in Fig. 7 and 8, the ACI limit (λ=22) for unbraced members is seen to be quite conservative at lower axial load levels. At higher axial load levels, on the other hand, it becomes unconservative, in particular for low reinforcement levels. However, even so, it is still below the 10% capacity reduction results in Fig. 7. Furthermore, compared to results for more non-uniform first order moment distributions that may be more realistic for sway members, Fig. 14 and 15, the ACI limit is in reasonably conformance with 5% capacity reduction results at higher axial load levels, but more conservative at lower levels.

The NS 3473 parameter λ_N differs from the present λ_n only due to differences in the chosen k_t–values. For the *RC* and *RD*–sections considered here, NS 3473 specifies k_t=4 and 2.67, respectively. In comparison, k_t=3.3 and 2.2 is used in the present presentation of results. Consequently, λ_N=10 will be slightly less conservative than λ_n=10.

The proposed EC2 limit, Case 9 in Table 1, is very similar to that in NS 3473 and the present study. This can be best seen by rewriting it as

$$\lambda\sqrt{\frac{\nu}{1+2\omega_t}} = \frac{14}{1+0.2\phi_{ef}} \tag{13}$$

Predictions are not included in the figures in order not to overcrowd them. In terms of λ_n for a *RD* section, with a k_t=2.2 that is approximately equal to the comparable value of 2.0 above, Eq. 13 gives λ_n=13.9 and 13.6, respectively, for ω_t=0.2 and 1.0, when creep is not included (ϕ_{ef}=0). These compare well with the 10% reduction results of the present nonlinear analysis. With creep of ϕ_{ef}=2 included, the corresponding predictions become λ_n=9.9 and 9.7, which corresponds about to the present 5% reduction results. For a *RC* section, with k_t=3.3, the predictions by Eq. 13 become more conservative relative to the nonlinear analysis results.

6 Sustained loading effects

Pertinent results from two studies, both obtained by non-linear analyses in combination with the rate of creep method for estimating long term deformations, are reviewed below. The short term material properties used was almost identical to those adopted in the present study (Fig. 6). No concrete strength increase, caused by aging effects (continued hydration following loading), is included in either study.

Fig. 16 shows time dependent results pertinent to criterion *a* (reduction in moment capacity under constant axial load). The results, derived by Aas-Jakobsen (1973; CEB 1977), are for a cantilever column with moderate reinforcement and moderate axial load (ν = 0.4, close to the balanced load). It has rectangular, corner reinforced cross-section, ε_y=0.0021 and h'/h = 0.8.

The column was subjected to a lateral top load that was sustained for a certain time period (t_s) with a creep factor ϕ_{ts}=2.2. Subsequently, it was loaded to failure by increasing the lateral load (see figure insert), and thus the moment. The axial load was maintained at its full value throughout, i.e., the sustained axial load was equal to the axial load at failure.

The slenderness limit, λ_n=10, is added to the figure (using k_t=3.93 as given by Eq. 8). At this limit, creep effects are seen to be completely negligible. This

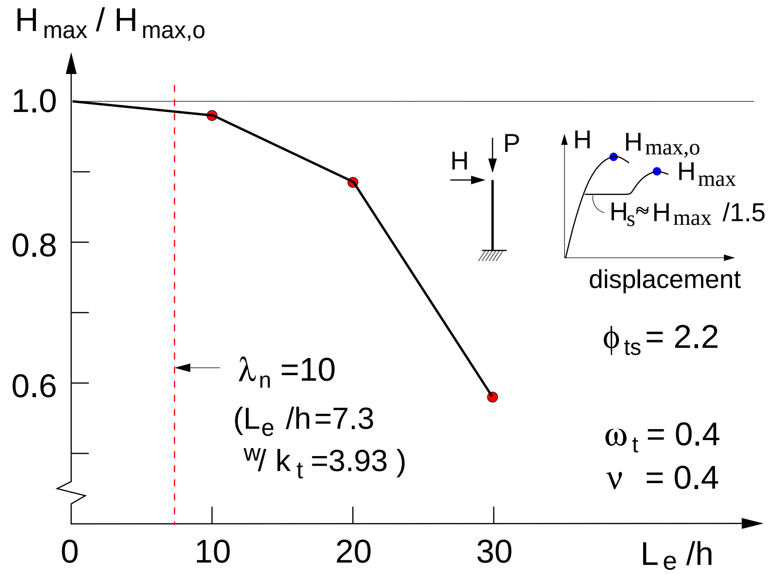


Figure 16: Effect of sustained loading on lateral load capacity

will also be the case at λ_n=12 (which will be proosed allows for triangular first order moment distributions). They will be small even for a higher creep factor. Further, the axial load history is rather severe and somewhat unrealistic. Also, the sustained lateral load, which is given to be about 1/1.5= 0.67 times the failure load, is roughly 60% of the nominal, non-slender column capacity. This is a high sustained level, at or possibly beyond a practical upper limit. Finally, had even a modest strength increase due to aging been included, it would have more than cancelled the creep effects at the considered limit, and even at larger values.

Creep effects increase with increasing axial load level. Some numerical results at very high axial loads that are pertinent to criterion c ("load capacity reduction for constant moment") have been reported by Manuel and MacGregor (1967). They studied the effect of sustained load periods on axial load capacities of columns in braced frames. The full beam loading ("moment loading") was applied first and kept constant throughout the load history. Axial loads, applied at column ends, were sustained at specified levels and subsequently incremented quickly to column failure. The creep coefficient for the sustained duration (t_s=25 years) was about ϕ_{ts}=3.5 to 3.8 in the linear creep range. Of interest here are results in that study for columns in single, uniform first order curvature. Unlike the column in Fig. 5(a1), the actual columns were rotationally restrained by identical beams at column ends.

From the results for these columns, it can be deduced that axial load strengths of $0.95N_d$ can be developed for slendernesses less than about $\lambda_n = 40$ ($L/h = 22$, elastic $\beta = 0.80$, $\lambda = 61$) for short term loading, and less than $\lambda_n = 23$ ($L/h = 12$, elastic $\beta = 0.87$ corresponding to a G–factor of G=2.3 at each end, and $\lambda = 36$)

for columns with a sustained load level of N_s/N_d=0.6. The nominal (non-slender, short term) load eccentricity for the latter column was about e_0/h=0.09. This corresponds to an axial load very close to an axial load limit of $0.8(1+\omega_t)$.

Although creep significantly affected results at this small eccentricity, the proposed lower limit of λ_n=10 is still seen to be very conservative for this case. More so than the difference between criterion *a* and criterion *c* in Fig. 2 indicates. That this is so despite the very high creep coefficient used by Manuel and MacGregor, must be due to the end restraints. Although these are not particularly stiff, a transfer of moment from the column to the restraining beams take place (during the sustained loading and axial load incrementation). This moment relief tends, in isolation, to strengthen the column relative to an unrestrained column. In similar unrestrained members, without a potential for moment transfer, the difference to the proposed limit will become less than found above. The study is presented in more detail in Hellesland (2002b).

This review of sustained load effects is not extensive. However, all considered, it is felt that the presented results confirm the premise that it is acceptable to neglect creep at low and moderate load levels in combination with criterion *a* (reduction in moment capacity under constant axial load), and that criterion *a* is sufficiently conservative at higher axial load levels to allow for creep effects relative to the "constant moment" criterion *c* with creep included, and even more so in combination with the "constant eccentricity" criterion *b*.

The conclusion above can be corroborated by results in a recent, not yet published study (Mari and Hellesland 2002, report in preparation) on restrained and unrestrained columns. Preliminary results of the study are given in the companion report (Hellesland 2002b). An earlier study by Menegotto (1983) on eccentrically loaded columns (uniform first moment), and previously mentioned, allows a similar conclusion.

7 Proposal

Comments

As a basis for a lower limit proposal, the uniform moment case is considered too unfavourable (Fig. 14 and 15). Even a small gradient causes a marked lift above the uniform case. The slightly non-uniform moment distribution ("0.8 to 1.0") is considered to be a sufficiently conservative lower limiting case. For the *RC*–section, Fig. 10 shows that a reasonable approximation of the 5% capacity reduction results is given by λ_n=10. At 10% capacity reduction, λ_n=14 represents a good approximation.

These approximations are not equally good at lower axial load levels for the RD–section, as seen in Fig. 11, but still acceptable for practical axial load levels. Also, had tension stiffening effects been included in the computations, the slenderness results would be increased somewhat, in particular for the lower load levels in question. Further, it should be recalled that use of one set of partial safety factors along the member, both in and outside the critical region, is conservative. With differentiated factors, results would be lifted somewhat.

If considered desirable to make the limit formulation more safe at lower load levels, this can be accomplished in a simple manner by choosing an alternative slenderness formulation in which the axial load level is incremented by a small addition in the order of $\Delta\nu$=0.03 to 0.05 (Eq. 14). This slenderness parameter will be more conservative at all load levels, but in particular at lower axial load levels. It is similar to an earlier proposal (Hellesland 1993). Use of the alternative slenderness is illustrated for $\Delta\nu$=0.05 in Appendix A.

The proposal below is related to 5% capacity reduction. Based on previous discussion and the section on sustained loading, it is proposed not to include creep as an explicit parameter in the formulation.

Comparable limits related to 10% reductions can be obtained by increasing the "5% limits" by about 40% in the absence of creep. With creep effects, a smaller increase is recommended as creep effects will be somewhat greater at the 10% capacity reduction limit than at the 5% limit. Although this has not been checked in any detail, it is recommended in general to take the "10% limits" not greater than about 1.2-1.3 times the "5% limits".

Lower limits (5% capacity reduction)

For compression members not braced against sidesway, and for braced compression members with transverse loads between their ends, it is permitted to ignore slenderness effects

A. if $\lambda_n \leq 10$ in the general case;

B. if $\lambda_n \leq 12$ for members with significantly non-uniform first order moment distributions (triangular etc. for single or double curvature);

C. if $\lambda_n \leq 14$ for cantilevers subjected primarily to distributed wind loading.

The normalised slenderness λ_n is defined by Eq. 5 with k_t given by Eq. 8, or alternatively, with $k_t\omega_t$ defined by Eq. 9. (Acceptable simplifications of k_t are defined by Eq. 10).

The normalised slenderness can alternatively be defined more conservatively by taking λ_n=λ_{n+}, where

$$\lambda_{n+} = \lambda\sqrt{\frac{\nu + \Delta\nu}{1 + k_t\omega_t}} \tag{14}$$

The load increment can be chosen in the range $\Delta\nu$=0.03-0.05.

8 Summary and conclusions

Several factors affecting lower slenderness limits, some of which have not previously been considered in any depth, have been discussed and consequences clarified. Results of extensive nonlinear analyses are presented.

By including axial force and reinforcement in the slenderness parameter in addition to the conventional geometrical slenderness, a more rational and reliable slenderness measure is obtained. In terms of this "normalised" slenderness parameter, simple lower limits are proposed for unbraced (sway permitted) compression members as well as for braced members with transverse loads between ends. They are considered to be applicable for all concrete qualities, including LWA concrete.

The proposed slenderness limit formulation for the general case is slightly more conservative than that in NS 3473. With the additional limits for special cases (triangular first order moment distribution etc.) it will still allow member slenderness effects to be ignored in many more cases, it is believed, than allowed by NS 3473 and other current limits. It is felt that reduced design efforts due to this, and the added reliability of the formulation, more than compensates for the added complexity of using a normalised slenderness than only a geometrical slenderness. The "complexity" is still rather modest. The inclusion of the additional parameters also has the advantage of adding focus on and awareness of parameters that after all is important in design of compression members.

At a preliminary design stage, conservative assumptions, such as minimum reinforcement and axial load due to an unfavourable load case, maybe introduced. If necessary, checking with more refined values can be performed at a more advanced stage in the design process. Conservative assumptions may also be introduced into the present proposal for the purpose of obtaining simplified limit formulations if that should be considered desirable.

ACKNOWLEDGEMENTS

During and after graduate study (master equivalent) at the University of Oslo, Norway, Jan Aasrum assisted in providing part of the numerical data on which this study is based. This assistance and the support provided by the Norwegian Research Council during research leaves in the fall of 1999 (Univ. of Texas at Austin) and 2001 (Universitat Politecnica de Catalunya, Barcelona) are greatly appreciated.

NOTATION

A_c = area of concrete section; can normally be taken as A_g
A_g, A_{st} = area of gross section and of total reinforcing steel
I_g, I_s = second moment of area about centroidal axis of gross section and of total reinforcing steel
L = length of compression member
L_e = effective (buckling) length of compression member
M_d, M_f = design moment capacity and factored 1st order moment ("ultimate")
M_{0f} = factored 1st order moment ("ultimate")
N_d, N_f= design axial load capacity and factored axial load ("ultimate"), positive as compressive
$f_{cd} = f_{cn}/\gamma_c$ = design compressive strength of concrete (NS 3473)
$f_{cd} = \alpha_{cc} f_{ck}/\gamma_c$ = design compressive strength (EC2, draft 2002)
of concrete (NS 3473)
f_{ck} = concrete **cylinder** compressive strength
f_{cn} = nominal structural (in situ) compressive strength of concrete
f_y = yield strength of reinforcing steel
$f_{yd} = f_y/\gamma_s$ = design yield strength of reinforcing steel
γ_c, γ_s = material factors for concrete and reinforcing steel
h, h' = section depth and distance between reinforcement in opposite faces
i = radius of gyration of cross section, normally taken as i_g
i_g, i_s = radii of gyration of gross concrete section and of total reinforcing steel.
β = effective (or buckling) length factor of compression member
ϕ_{ts} = creep coefficient for time period t_s
γ_c, γ_s = partial safety factors of concrete and steel
ω_t = total mechanical reinforcement ratio
ν = relative factored axial load
ε, σ = strain and stress
Subscript t = total
Subscript d = design
Subscript k = characteristic
Subscript y = yield

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Appendix A – Alternative slenderness parameter

As mentioned in the main text, an alternative to Eq. 8 that has been considered in previous studies is

$$k_t = \frac{5.2}{1000\,\varepsilon_{yd}} \left(\frac{i_s}{i_g}\right)^2 \tag{15}$$

With this expression, k_t=4 is obtained for a RC section with h'/h = 0.8 and ε_{yd} = 0.0025. In comparison, Eq. 8 gave a value of k_t=3.3 for the same case. Results for these two values of k_t were compared in Fig. 9.

The difference between the two sets of results is partly different gradients of the curves, which, however, is not that easy to observe from the figure. The difference that is most obvious is that the use of the greater value k_t=4 (Eq. 15) lowers the curves in comparison with the curves obtained using k_t=3.3 (Eq. 8).

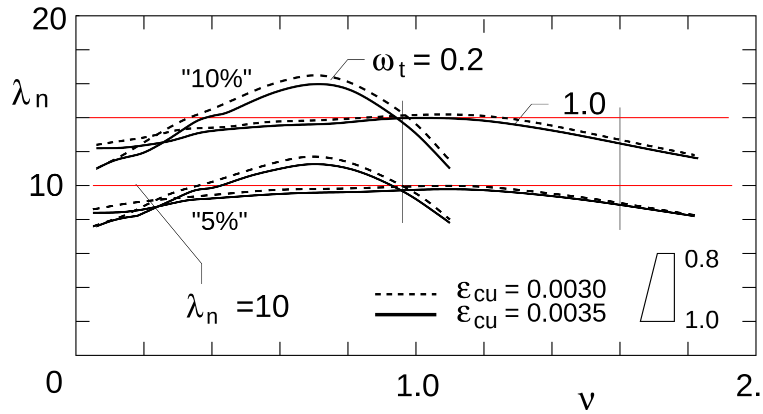

Figure 17: Normalised slenderness vs. axial load level for two ultimate concrete strains (k_t=4)

Fig. 10 (k_t=3.3), is replotted in Fig. 17 with k_t=4. The same lowering of curves is seen here. By arguments such as neglect of tension stiffening and that the same material factors, applicable to the critical zone, are used for the whole member, may allow the conclusion that the results in Fig. 17 still justifies a lower limit of λ_n=10 related to a 5% capacity reduction. Similarly, λ_n=14 would be reasonable for 10% capacity reduction.

However, if desired, more conservative results can be obtained with k_t given by Eq. 15 by lowering the limit below 10 (14).

Alternatively, it is possible with rather easy means to adjust the curves upwards, and in particular so at low axial loads where this is of most interest, through the use of an alternative normalised slenderness definition. It has been considered previously (Hellesland 1993) and is defined by Eq. 14. With $\Delta\nu$=0.05, it becomes

$$\lambda_{n+} = \lambda \sqrt{\frac{\nu + 0.05}{1 + k_t \omega_t}} \tag{16}$$

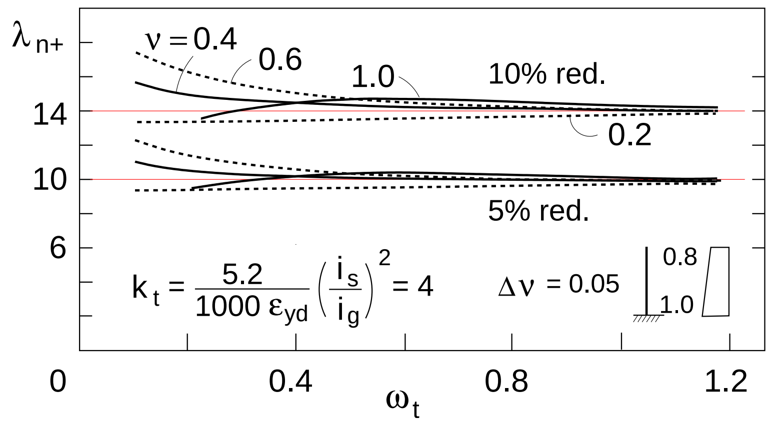

Figure 18: Alternative normalised slenderness vs. reinforcement for different axial load levels ($k_t = 4$)

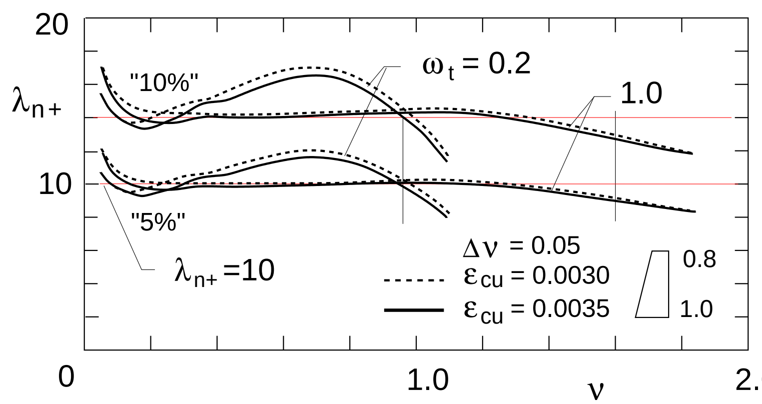

Figure 19: Alternative normalised slenderness vs. axial load level for two ultimate concrete strains

Fig. 9, upper part (k_t=4), is redrawn in Fig. 18 in terms of Eq. 16 with k_t=4 (Eq. 15). The inclusion of the small addition (0.05) to the relative axial force in the slenderness parameter is very effective in bringing the lines for the different axial forces relatively close together. Fig. 18 also includes results for 10% capacity reduction (located about 40% above those for 5% reduction).

Fig. 17 above is redrawn in Fig. 19 in terms of the alternative slenderness. The lifting of the curves at low axial loads is most notable.

The k_t above is 1.209 (=5.2/4.3) times the one adopted in Eq. 8 and used in presentations in the main part of this study. Clearly, the alternative normalised slenderness may also be used in conjunction with k_t given by Eq. 8. That would lift the curves in Fig. 19 upwards and call for limiting values increased beyond 10 (14) if added conservativeness is not desired.

Appendix B – Nonlinear analysis details

The differential equation, $v'' = -\kappa$, where v'' is the second derivative of the displacement, $\kappa = 1/R$ the curvature and R the radius of curvature, was discretised using central differences. To suit the present application it is rewritten as

$$v_{i+1} = 2v_i - v_{i-1} - \kappa_i(\Delta x)^2 \qquad i = 1, 2, \cdots, m-1 \tag{17}$$

and applies to a member divided into m elements of equal length (Δx). Section $i = 0$ is at the base (clamped end) and $i = m$ at the free end of the cantilever. Eq. 17 allows extrapolation from one section to the next. At $i = 0$, $v_0 = 0$ (forced boundary condition) and the symmetry condition $v_{-1} = v_1$ (from $v' = (v_{i+1} - v_{i-1})/2\Delta x$) are introduced.

A. Determine slenderness limits for cantilever

The maximum moment will be located at section $i = 0$ (at the clamped end). The total moment value (including second order effects) at this end, $M_{t,0}$, will be between the cross-sectional moment capacity, M_d, and the required first order moment capacity, M_{1d}. For a chosen $M_{t,0}$, the free end displacement, a, is given by equilibrium. The member length L giving this a can be determined by iteration using Eq. 17. The value of $M_{t,0}$ for which L becomes the largest possible value corresponds to member instability. This L-value is the one to be determined. The description below gives the main steps. Details are given in Aasrum (1992). The program is very fast, and a fine subdivision is adopted (normally $\Delta x = h/30$).

1. Choose cross–section, reinforcement and axial load ($N_d = N_f$), and compute the moment–curvature relationship ($M_{d\kappa} - \kappa$) of the section (using an existing computer routine (by Hellesland)). The cross-sectional moment capacity, M_d, is the maximum $M_{d\kappa}$, and is obtained at predefined ultimate strains (ε_{cu}, or $\varepsilon_{su} = 0.010$ if it gives lower moment resistance than ε_{cu}).

2. Specify required first order moment capacity M_{1n} (in this study equal to $0.95M_d$ or $0.9M_d$, corresponding to capacity reductions of 5 or 10%) .

3. Choose total end moment $M_{t,0}$ (between M_d and M_{1d}) and determine the member length L that corresponds to the chosen $M_{t,0}$ by iteration as described below in routine B.

4. Repeat from Step 3 with a new $M_{t,0}$ until the largest possible L-value, L_{max}, is obtained. (By starting off in Step 3 with $M_{t,0}$ equal or close to M_d, and then gradually decreasing it (towards M_{1d}) in each iteration, the resulting L-values initially increase towards a maximum (L_{max}) and then start decreasing.

5. Set $L = L_{max}$ and compute the slenderness parameters $\lambda = L_e/i_g$ or $\lambda_n = \lambda\sqrt{(\nu/(1 + k_t\omega_t)}$. The adopted effective length is taken equal to the elastic effective length of a member with constant EI, i.e., $k = 2$ for the member clamped at one end and free at the other.

6. Repeat from step 1 or 2 (as desired).

B. Calculation of L, iteration scheme

1. Calculate the end displacement from equilibrium: $a = (M_{t,0} - M_{1d})/N_d$.

2. Assume L. (A reasonable start value in the first iteration is given by $L = 0.5\sqrt{10a/\kappa_0}$. Here, κ_0 is the curvature in the $M_{d\kappa} - \kappa$ – relationship that corresponds to the chosen moment $M_{t,0}$).

3. Compute external loads (H, M_B, q) that give the chosen moment distribution and specified first order moment M_{1d} at the base. Examples: $H = M_{1d}/L$ if only H is acting, $q = 2M_{1d}/L^2$ if only q is acting, etc.

4. At all discrete sections, compute first order moments $M_{0,i}$ (for external loads) and total moments $M_{t,i} = M_{0,i} + N_d(a - v_i)$, with $v_i = 0$ in the first iteration, and determine corresponding curvature values from the $M_{d\kappa} - \kappa$ relationship in step A1.

5. Compute displacements at all discrete sections using Eq. 17, and repeat from Step 2 with a new L until there is no significant difference between v_m (at section $i = m$) and a (Step 1). (An efficient approach is to set $L_{new} = L_{old}\sqrt{a/v_m}$ and to iterate until there is no significant difference in L-values from two consecutive iterations (adopted tolerance 0.1%)).