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Investigating the predicted breathing-mode excitation of the Hoyle state

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ABSTRACT

Knowledge of the low-lying monopole strength in ^{12}C—the Hoyle state in particular—is crucial for our understanding of both the astrophysically important 3α reaction and of α-particle clustering. Multiple theoretical models have predicted a breathing mode of the Hoyle State at $E_x \approx 9$ MeV, corresponding to a radial in-phase oscillation of the underlying α clusters. The $^{12}\mathrm{C}(\alpha,\alpha')^{12}\mathrm{C}$ and $^{14}\mathrm{C}(p,t)^{12}\mathrm{C}$ reactions were employed to populate states in ^{12}C in order to search for this predicted breathing mode. A self-consistent, simultaneous analysis of the inclusive spectra with **R**-matrix lineshapes, together with angular distributions of charged-particle decay, yielded clear evidence for excess monopole strength at $E_x \approx 9$ MeV which is highly collective. Reproduction of the experimentally observed inclusive yields using a fit, with consistent population ratios for the various broad states, required an additional source of monopole strength. The interpretation of this additional monopole resonance as the breathing-mode excitation of the Hoyle state would provide evidence supporting a $\mathcal{D}_{3h}$ symmetry for the Hoyle state itself. The excess monopole strength may complicate analysis of the properties of the Hoyle state, modifying the temperature dependence of the 3α rate at $T_9 \gtrsim 2$ and ultimately, the predicted nucleosynthesis in explosive stars.

The emergent phenomenon of α-particle clustering in light nuclei has garnered significant interest for both nuclear structure and astrophysics, resulting in predictions such as dilute densities and Bose-Einstein condensation [1,2]. Of particular interest is ^{12}C, which exhibits shell-model and α-cluster structures. The Hoyle state in ^{12}C is the archetypal α-cluster state [3], remaining the focus of considerable study, including efforts to measure its direct [4–9], γ [10] and $E0$ decay branching ratios [11]. Such decay branches are astrophysically significant; the Hoyle state mediates the 3α reaction producing ^{12}C. The 3α reaction rate requires an accurate description of the Hoyle-state properties (e.g., resonance energy E_r, partial and total widths, Γ_i and Γ) and their evolution with excitation energy. In the region $0.1 < T_9 < 2$ ($T_9 = T/10^9$ K), the 3α rate is determined by the Q value, and the γ-ray and pair-production partial widths [12]. At $T_9 > 2$, higher-lying resonances, such as the rotational 2^+ excitation of the Hoyle state [13–15],

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and the Hoyle state's "ghost" contribute to the 3α reaction rate [12]. The ghost of the Hoyle state corresponds to a pronounced high-energy tail with a local maximum above the narrow primary peak of the Hoyle state. This phenomenon results from the strong α-cluster character of the Hoyle state and its proximity to the α-separation energy [3,16]. Understanding the evolution of the Hoyle state's properties (e.g. partial widths and branching ratios) with excitation energy, and the number and nature of the additional excited levels above the Hoyle state, is vital in computing the 3α rate in explosive burning. The partial widths of the Hoyle state, and subsequently the 3α reaction, may be inaccurately estimated if there are unaccounted-for sources of monopole strength or if the phenomenological description of the Hoyle state is insufficient, failing to properly describe the observed monopole strength.

Multiple independent predictions of the Hoyle-state breathing-mode monopole strength with a variety of theoretical models have been made in recent years [17–22]. This breathing-mode excitation corresponds to a radial in-phase oscillation of the underlying α clusters. This excitation is predicted to lie at $E_x \approx 8 - 9.5$ MeV between two previously established sources of monopole strength: the 0_2^+ Hoyle state at $E_x = 7.65407(19)$ MeV with its associated ghost and a broad 0_3^+ state at $E_x \approx 10 - 11$ MeV [23–25] with $\Gamma \approx 3$ MeV [25]. Some models [26,27,21,20] also predict an additional source of monopole strength above $E_x \approx 10$ MeV, which may correspond to the previously observed 0_3^+ state.

A novel interpretation of the structure of ^{12}C with $\mathcal{D}_{3h}$ symmetry, historically employed to study triatomic molecules, provided a compelling description of high-spin states as the rotational excitations of an equilateral triangular structure [28–30]. A breathing-mode excitation of a 3α cluster state, depicted as the radial in-phase oscillation of the component α clusters, corresponds to a characteristic vibrational mode of $\mathcal{D}_{3h}$ symmetry. The existence of such a breathing-mode excitation, specifically built on the Hoyle state, would suggest $\mathcal{D}_{3h}$ symmetry for the Hoyle state itself. This is in contrast to some studies which suggest a "bent-arm" (obtuse-triangle) structure for the Hoyle state, resembling $^8\mathrm{Be} + \alpha$ configurations [26,31]. Given the role of the Hoyle state in understanding α clustering, determining if it has a breathing-mode excitation is of great importance.

Identification of this predicted breathing-mode excitation is complicated by phenomenological and experimental factors. The ghost of the Hoyle state extends some considerable energy above the main peak of the Hoyle state, overlying the region in which the breathing-mode is predicted and resulting in interference effects with higher-lying monopole resonances [32]. The region above the Hoyle state was studied by Itoh et al. through the $^{12}\mathrm{C}(\alpha,\alpha')^{12}\mathrm{C}$ reaction and a peak-fitting analysis with Gaussian lineshapes required an additional peak at $E_x \approx 9.04(9)$ MeV with $\Gamma = 1.45(18)$ MeV [13]. However, the Gaussian lineshapes employed in Ref. [13] do not capture the behavior of near-threshold resonances or interference effects. Since a multipole decomposition analysis (MDA) revealed the region at $E_x \approx 9$ MeV to be predominantly monopole, a number of authors [33,21,19,34] have discussed the additional Gaussian peak in the context of the breathing-mode excitation of the Hoyle state. A more detailed analysis is required, taking into account the complex shape of the Hoyle state and potential interference effects between resonances. The objective of this work is to investigate sources of monopole strength in ^{12}C between $E_x = 7$ and 13 MeV using a consistent analysis of $^{12}\mathrm{C}(\alpha,\alpha')^{12}\mathrm{C}$ and $^{14}\mathrm{C}(p,t)^{12}\mathrm{C}$ reaction data. The intention is to determine if the two previously established sources of monopole strength and the interference between them can reproduce the experimental data, or if an additional source of monopole strength is required.

To populate the excitation-energy region of interest, measurements of the $^{12}\mathrm{C}(\alpha,\alpha')^{12}\mathrm{C}$ and $^{14}\mathrm{C}(p,t)^{12}\mathrm{C}$ reactions at divers laboratory angles and beam energies were performed using the K600 spectrometer at the iThemba Laboratory for Accelerator-Based Sciences (iThemba LABS) in South Africa. The experimental conditions are summarized in Table 1 and a comprehensive description is reported in Ref. [35]. Measurements with different selectivity were chosen to exploit differences in population strength between contributing broad states, and enable a self-consistent, simultaneous analysis of the inclusive spectra. The $^{12}\mathrm{C}(\alpha,\alpha')^{12}\mathrm{C}$ reaction at $\theta_{\mathrm{lab}} = 0°$ strongly populates collective monopole excitations, in contrast to the $^{14}\mathrm{C}(p,t)^{12}\mathrm{C}$ reaction which is less selective to collective isoscalar monopole excitations [36–38]. Proton and α-particle beams were extracted from the separated-sector cyclotron and transported down a dispersion-matched beamline to the target position of the K600 magnetic spectrometer [39]. Ejectiles were momentum-analyzed in the spectrometer and detected at the focal plane in a modular combination of vertical drift chambers and plastic scintillators. Coincident charged-particle decay from excited ^{12}C states were detected in the CAKE, an array of double-sided silicon strip detectors [40], for the measurements of $^{12}\mathrm{C}(\alpha,\alpha')^{12}\mathrm{C}$ at $\theta_{\mathrm{lab}} = 0°$ (E_α = 200 MeV) and $^{14}\mathrm{C}(p,t)^{12}\mathrm{C}$ $\theta_{\mathrm{lab}} = 0°$.

Table 1
Summary of experimental parameters.

Reaction	Angle [deg]	E_{beam} [MeV]	Target (μg/cm^2)	Fitted E_x range [MeV]
$^{12}\mathrm{C}(\alpha,\alpha')^{12}\mathrm{C}$	0	118	$^{\mathrm{nat}}$C (1053)	5.0 - 14.8
	0	160	$^{\mathrm{nat}}$C (300)	7.3 - 20.0
	0	200	$^{\mathrm{nat}}$C (290)	N.A.[a]
	10	196	$^{\mathrm{nat}}$C (290)	7.15 - 21.5
$^{14}\mathrm{C}(p,t)^{12}\mathrm{C}$	0	100	^{14}C (280)	6.0 - 15.3
	21	67.5	^{14}C (300)	6.8 - 14.5

[a] Only the charged-particle decays were analyzed as the Hoyle state was not fully accepted on the focal plane.

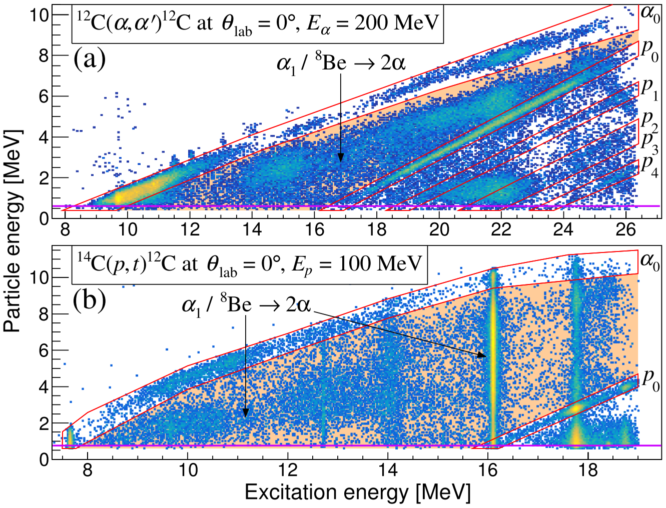

Fig. 1. Matrices of decay-particle energy versus the excitation energy of ^{12}C. Red lines indicate the observed α_0 and proton decay modes of ^{12}C. The excitation energy of the ^{12}C recoil is inferred from the momentum of the α/triton ejectile measured at the focal plane. α_0 and α_1 denote α decay to the ground and first-excited states of ^{8}Be, respectively. p_0 and p_1 denote proton decay to the ground and first-excited states of ^{11}B, respectively (and so forth). Loci within the orange-highlighted regions correspond to α particles from either the α_1 decay of ^{12}C or $^8\mathrm{Be} \to 2\alpha$ breakup. Violet, horizontal lines indicate the approximate electronic thresholds for the CAKE.

The matrices of decay-particle energy (detected in CAKE) vs. excitation energy are shown in Fig. 1: loci corresponding to the α and proton (p) decay modes are observed, with α_0 and α_1 denoting α decay to the ground and first-excited states of ^{8}Be, respectively.

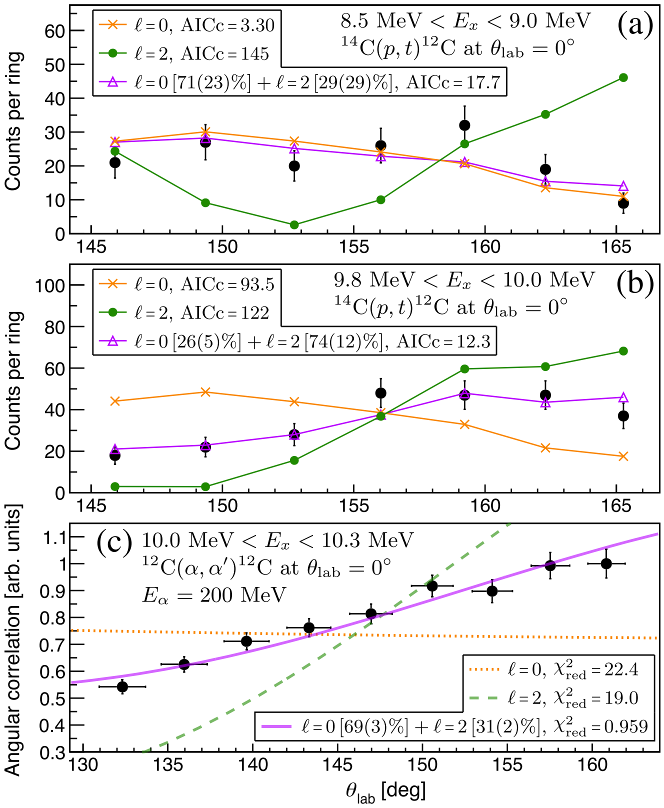

Fig. 2. Angular correlations of α_0 decay obtained in the ^{12}C$(\alpha,\alpha')^{12}$C and ^{14}C$(p,t)^{12}$C reactions at bombarding energies of 100 and 200 MeV, respectively. For distributions with low counts, the AICc estimator was used to determine the best quality model [41,42].

For $E_x = 8.5$ to 9.0 MeV populated with the ^{14}C$(p,t)^{12}$C reaction at $\theta_{\text{lab}} = 0°$, the α_0 angular correlation in Fig. 2(a) is predominantly described by isotropic decay. This is consistent with Ref. [13] indicating that this region is dominated by monopole strength. For the ^{12}C$(\alpha,\alpha')^{12}$C* reaction, the α_0 decay cannot be reliably measured below $E_x \approx 10$ MeV since the α particles fall below the threshold of the silicon detectors (see Fig. 1). For the ^{14}C$(p,t)^{12}$C reaction, the momentum boost of the recoiling ^{12}C nucleus means that α particles have a higher laboratory energy and can be detected down to a lower corresponding excitation energy.

Figs. 2(b) and 2(c) show the angular correlations for α_0 decay in the regions $E_x = 9.8$ to 10.0 MeV and $E_x = 10.0$ to 10.3 MeV for the ^{14}C$(p,t)^{12}$C and ^{12}C$(\alpha,\alpha')^{12}$C reactions, respectively. For both cases, the data are well reproduced by an incoherent sum of $\ell = 0$ and $\ell = 2$ decay components which are of the same order of magnitude. This result confirms the existence of the 2^+_2 state in agreement with the ^{12}C$(\alpha,\alpha')^{12}$C measurements of Itoh et al. [13] and Freer et al. [14]. In summary, the angular correlations show that the $E_x \approx 9$ MeV region is primarily monopolar in nature, and that a consistent description of the $E_x \approx 10$ MeV region must include the 2^+_2 state with a strength comparable to the broad monopole contributions at $E_x \approx 10$ MeV for the ^{14}C$(p,t)^{12}$C and ^{12}C$(\alpha,\alpha')^{12}$C reactions at $\theta_{\text{lab}} = 0°$.

The inclusive excitation-energy spectra from five different measurements (see Table 1) were simultaneously analyzed with phenomenological lineshape parameterizations from multi-level, multi-channel **R**-matrix theory. Only the α_0 and α_1 decay modes are considered; the proton-decay channel is not open at the excitation energies of interest for this work. For α_1 decay, the penetrability accounts for the broad width of the first-excited 2^+_1 state of ^{8}Be. For decays with multiple possible angular momenta of decay, the lowest ℓ-value of decay is assumed to dominate and is set as the exclusive channel. Instrumental backgrounds were simultaneously fitted. Experimental factors, such as the spectrometer ion optics and the response of the drift chambers, were included in the analyses of the spectra [35]. Feeding factors capturing the excitation-energy dependence for each incoming reaction channel were determined with CHUCK3 [43]. Fig. 3 presents the optimized **R**-matrix fits, which only include the previously established resonances between $E_x = 7$ and 13 MeV (as well as contaminant peaks), where the Hoyle-state width has been fixed at $\Gamma = 9.3$ eV, for the measurements of ^{12}C$(\alpha,\alpha')^{12}$C at $\theta_{\text{lab}} = 0°$—a reaction that is highly selective for collective monopole strengths. Including only the Hoyle state provides a poor description of the data, with a large underestimation of the experimental spectra at $E_x \approx 9$ MeV. Similarly, the models including both the 0^+_2 and 0^+_3 states without interference and with destructive α_0 interference (denoted $-\alpha_0$) underestimate the data at $E_x \approx 9$ MeV. The optimal model with all previously established states is achieved by including both the 0^+_2 and 0^+_3 monopole resonances with constructive α_0 interference (denoted $+\alpha_0$), however the systematic underestimation of the data at $E_x \approx 9$ MeV remains. A decomposition of the best fit, shown in Fig. 4, indicates a highly suppressed strength for the 2^+_2 state located at $E_x = 9.870(60)$ MeV. This is inconsistent with the charged-particle decay data discussed earlier and shown in Fig. 2, which indicate significant 2^+_2 strength at $E_x \approx 10$ MeV, as well as the analyses of ^{12}C$(\alpha,\alpha')^{12}$C presented in Refs. [13,14]. To test whether a difference in the total width of the Hoyle state may improve the fit, the total width of the Hoyle state was tested at 5σ below and above the listed value of $\Gamma = 9.3(9)$ eV [25], with the 5σ band in Fig. 3 corresponding to the range spanned by the associated fits. Even at these extreme values, a clear systematic excess in the data remains in the dominantly monopole region at $E_x \approx 9$ MeV.

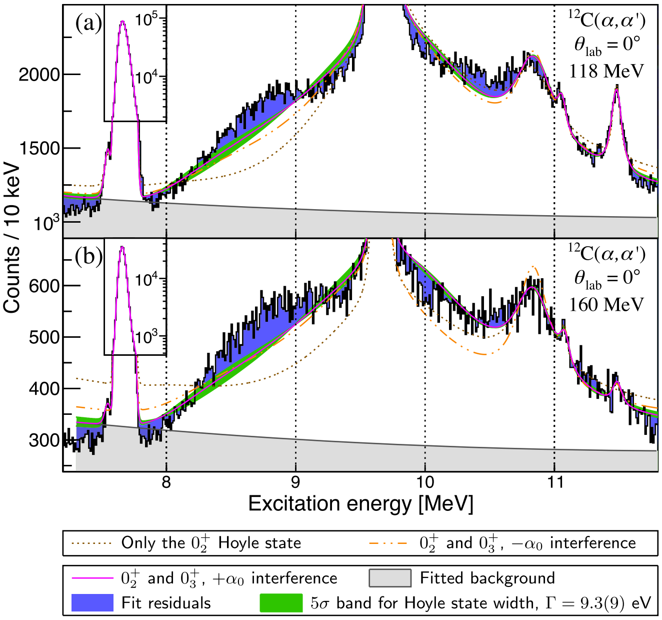

Fig. 3. Comparison of fits, using only previously established resonances, for the inclusive inelastically scattered alpha yields of the ^{12}C$(\alpha,\alpha')^{12}$C reaction at $\theta_{\text{lab}} = 0°$ with different beam energies.

To account for the excess strength at $E_x \approx 9$ MeV, which according to both the charged-particle decay data of this work and Ref. [13] is monopolar in nature, an additional monopole state was introduced, denoted 0^+_Δ. Two parameterizations for the resulting monopole strength were investigated: in the first, the 0^+_Δ state was permitted to interfere with the previously established

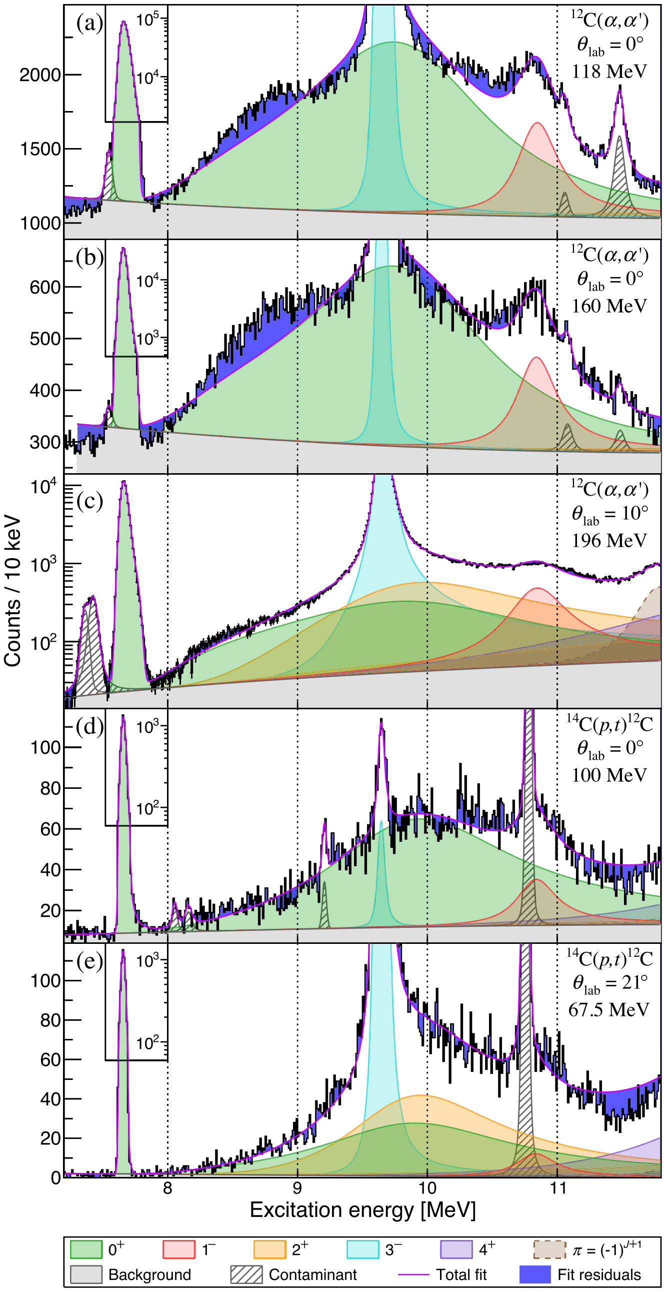

Fig. 4. Decomposition of the optimal fit which only includes previously established resonances: the 0_2^+ and 0_3^+ states constructively interfere in the α_0 channel (see Fig. 3). Panels (a) - (e) correspond to the inclusive ejectile yields for the employed $^{12}\mathrm{C}(\alpha,\alpha')^{12}\mathrm{C}$ and $^{14}\mathrm{C}(p,t)^{12}\mathrm{C}$ reactions at various measurement angles and beam energies.

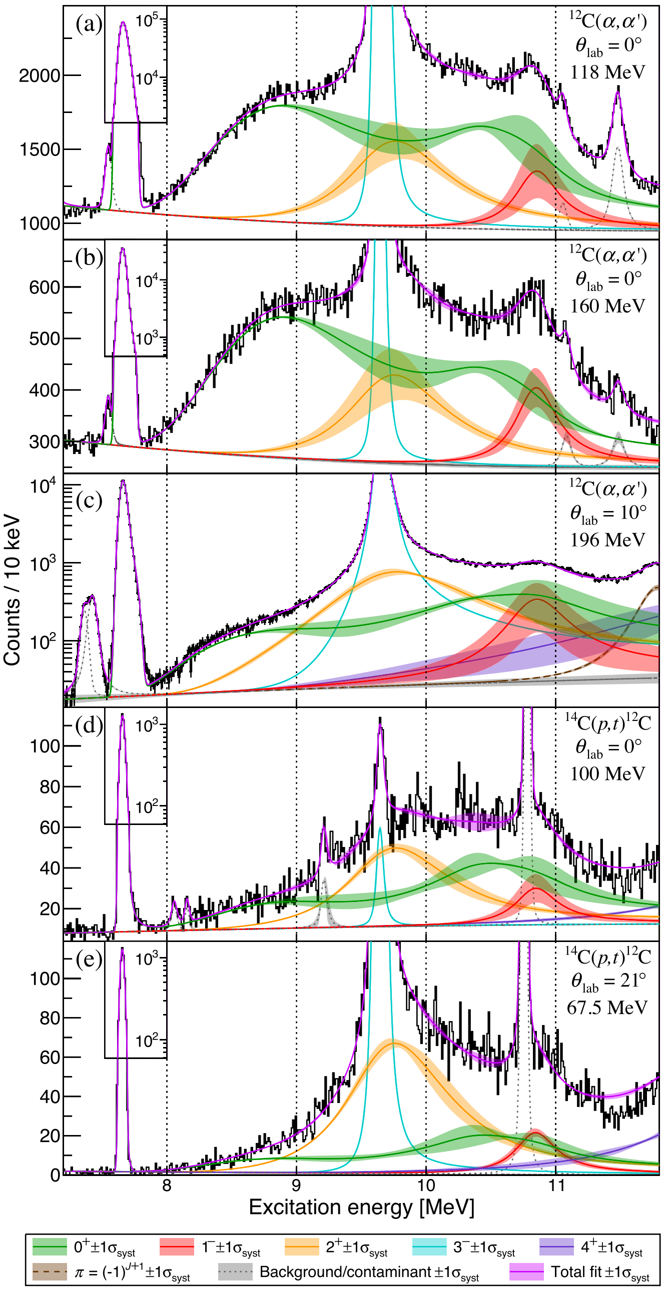

Fig. 5. Decomposition of the optimal fit which includes the previously established 0_2^+ and 0_3^+ states as well as the additional 0_Δ^+ resonance at $E_x \approx 9$ MeV. Panels (a) - (e) correspond to the inclusive ejectile yields for the employed $^{12}\mathrm{C}(\alpha,\alpha')^{12}\mathrm{C}$ and $^{14}\mathrm{C}(p,t)^{12}\mathrm{C}$ reactions at various measurement angles and beam energies.

0_2^+ and 0_3^+ states. In the second, interference involving the 0_Δ^+ state was omitted whilst the 0_2^+ and 0_3^+ states were still permitted to interfere. The optimized fit for the first parameterization is presented in Fig. 5, which produces a significantly improved description of the data, particularly at $E_x \approx 9$ MeV. The 1σ bands on Fig. 5 show the systematic dispersion of lineshapes corresponding to different permutations of interference and channel-radii. The decomposition of the fit reveals contributions from the 2_2^+ state which are comparable to the broad monopole strength at $E_x \approx 10$ MeV for the measurements of $^{12}\mathrm{C}(\alpha,\alpha')^{12}\mathrm{C}$ and $^{14}\mathrm{C}(p,t)^{12}\mathrm{C}$ at $\theta_{\mathrm{lab}} = 0°$—a result consistent with the present charged-particle decay data and Ref. [13]. Furthermore, the fit reveals the overall monopole strength to be double peaked in the region $E_x =$ 9 − 11 MeV for the $^{12}\mathrm{C}(\alpha,\alpha')^{12}\mathrm{C}$ reaction ($\theta_{\mathrm{lab}} = 0°$), in agreement with Ref. [13]. The additional 0_Δ^+ state has best fit parameters of $E_r = 9.566 \pm 0.018_{(\mathrm{stat})} \pm 0.104_{(\mathrm{syst})}$ MeV with $\Gamma(E_r) = 3.203 \pm 0.061_{(\mathrm{stat})} \pm 0.599_{(\mathrm{syst})}$ MeV. The second parameterization, which omitted possible interference involving the 0_Δ^+ state, yielded $E_r = 9.379 \pm 0.013_{(\mathrm{stat})} \pm 0.050_{(\mathrm{syst})}$ MeV with $\Gamma(E_r) = 4.565 \pm 0.022_{(\mathrm{stat})} \pm 0.107_{(\mathrm{syst})}$ MeV with $\Gamma_{\mathrm{FWHM}} = 2.066 \pm 0.005_{(\mathrm{stat})} \pm 0.098_{(\mathrm{syst})}$ MeV [35].

There is no consensus on the number and properties of the monopole states in this astrophysically important excitation-energy region. This is due to the variety of analysis methods used to describe the data, some of which do not include the strong threshold effects or interference. However, there is a rough similarity between the additional 0_Δ^+ state in this work to a structure in

Ref. [13] that was described with a Gaussian peak. Since the peak-fitting analysis with Gaussian lineshapes in Ref. [13] did not account for the physical effects of near-threshold resonances (such as the Hoyle state's ghost) or interference, care must be taken in this comparison since the two analyses are based on different foundations, resulting in different relative population strengths. Good agreement with recent measurements [32,44] of the 2_2^+ and 0_3^+ states was also achieved in this work (see Ref. [35] for details).

From the analysis above, there is clear evidence for an additional source of monopole strength in the excitation-energy region of $E_x = 7$ to 13 MeV; the data cannot be reproduced with only the previously established 0_2^+ and 0_3^+ states within the acceptable model space for the parameters of those states. The introduction of a new monopole state significantly improved the description of the data, including calculated relative strengths for the new monopole state and the 2_2^+ state in agreement with the angular correlations from charged-particle decay in the present work and the multipole decomposition analysis of Ref. [13]. The parameters of this new monopole state are in good agreement with theoretical predictions [17–19,21,22], making this state the leading candidate for the breathing-mode excitation of the Hoyle state.

That a significant excess of monopole strength is consistently observed at $E_x \approx 9$ MeV with $^{12}\mathrm{C}(\alpha,\alpha')^{12}\mathrm{C}$ at $\theta_{\mathrm{lab}} = 0°$, but not with $^{14}\mathrm{C}(p,t)^{12}\mathrm{C}$ at the same angle is suggestive of a collective character for this additional monopole state. The former reaction is considerably more selective for collective isoscalar monopole excitations than the latter [36–38]. The interpretation of the additional 0_Δ^+ state as a breathing-mode excitation is consistent with its weak population in beta-decay data [32], as β-decay is less favorable for transitions involving radial changes in comparison to other nuclear/electromagnetic probes [45,46]. More evidence that the newly observed state should be identified as the breathing-mode excitation of the Hoyle state comes from the relative ordering of resonance energies, with the predicted excitation energy around 2 MeV above the Hoyle state and below the higher-lying ($E_x \gtrsim 10$ MeV) broad monopole strengths which have been interpreted to correspond to 'bent-arm" ($^8\mathrm{Be}+\alpha$) configurations [26,31] and/or large-amplitude breathing-mode excitations [20].

It is possible that the excess monopole strength at $E_x \approx 9$ MeV can be alternatively explained by the artifacts of the current phenomenological descriptions of how the Hoyle-state properties evolve with excitation energy. Phenomenological **R**-matrix models employed in astrophysics typically assume that only two-body effects are important [12,47]. Whilst the direct 3α decay branch has been shown to be small at the primary peak of the Hoyle state [5–7,9], at higher excitation energies, the direct branch may be non-negligible due to the enhanced penetrability through the Coulomb barrier (see Ref. [8]). More sophisticated parameterizations of the monopole strength [48] should be explored. Inaccurate parameterizations may result in inaccurate extraction of the properties of states in $^{12}\mathrm{C}$, affecting the 3α reaction rate particularly above 2 GK in, for example, the shock front for type II supernovae [12,49,50]. In such environments, improved theoretical descriptions, including the additional 0_Δ^+ state, may modify the triple-α reaction rate. A quantitative estimation is beyond the scope of this work as knowledge on the radiative widths for the broad contributions above the Hoyle state is required but is not available from these data. The inclusive spectra studied in this work are most sensitive to the resonance energies and total widths of resonances, the latter of which are dominated by charged-particle decay channels in the excitation-energy region of interest between $E_x = 7$ and 13 MeV. The radiative width for the rotational 2_2^+ excitation of the Hoyle state — a crucial quantity for the triple-α process at $T_9 \sim 4$ — has been measured [15]. This state is not of a similar nature to the collective 0_Δ^+ state, therefore no useful information on the reduced transition probabilities can be extracted from this comparison.

In summary, despite the above discussed aspects requiring further investigation, a leading candidate for the breathing mode excitation of the Hoyle state has been observed in additional monopole strength in $^{12}\mathrm{C}$ at $E_x \approx 9$ MeV. The evidence for the breathing mode at this excitation energy favors the Hoyle state being an equilateral triangle.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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