Title: A Comparative Analysis of the Orientation to Mathematical Competency Acquisition in School Curricula in Germany, Korea, and Norway

Authors:

Nils Buchholtz[1], Oh Nam Kwon[2], Kyungwon Lee[3], Guri Nortvedt[4]

[1]University of Cologne [2]Seoul National University, [3]Graduate School of Seoul National University, [4]University of Oslo,

Corresponding author:

Prof. Dr. Nils Buchholtz
Universität zu Köln,
Mathematisch-Naturwissenschaftliche Fakultät
Institut für Mathematikdidaktik
Gronewaldstr. 2 (Gebäudeteil C, R. 2.306)
D-50931 Köln
Tel: +49-221-470-4750
Fax: +49-221-470-4985
E-Mail: Nils.Buchholtz@uni-koeln.de

Abstract

The orientation of school curricula for mathematics in many countries is based on the formulation of central competencies that students should learn in class. The idea of competency orientation takes up central mathematical ways of thinking and working, which should be taught in addition to the mathematical content in the classroom. By means of a comparative description of competence orientation in three countries, Germany, Korea, and Norway, we show how the educational policy idea of competence orientation has developed as an example. By examining the similarities and differences between the countries, it becomes clear which guiding ideas are linked to competence orientation, and how research in mathematics didactics can influence educational policy in the countries.

Keywords: Competence; Competencies, educational policy, international comparison; curricula

1. Introduction

In her many years as a professor of mathematics education at the University of Hamburg, Gabriele Kaiser has repeatedly promoted the further development of mathematics teaching in Hamburg schools. Driven by the conviction that mathematics teaching should be geared toward general education and make students mathematically fit for the demands of the future, Gabriele Kaiser has long advocated anchoring applications and references to reality at the curricular level and steering mathematics teaching toward a competence orientation. She advocates an approach that does not reduce mathematics education to the mere teaching of subject content and solution algorithms, but rather places mathematics in subject-specific contexts and puts its connection to students' real life at the center of learning. This gives process-related mathematical competencies such as reasoning, problem solving, and mathematical modelling a high significance as activities for learning and applying mathematics. In particular, within the German-speaking debate on mathematics education, this point of view is now and then exposed to the accusation that the teaching of deeper mathematical knowledge in the classroom would thereby be diluted in favor of the teaching of "less-mathematical" skills, especially also in central examinations and educational qualifications (Kaiser & Busse, 2014). In her academic career, Gabriele Kaiser has repeatedly defended a competence-oriented mathematics education against accusations in educational policy and public discourse, always arguing in a scientifically sound manner and keeping the best interests of students in mind (cf. for example Kaiser & Busse, 2014). Throughout her scientific career, she has thus achieved not only a major scientific outreach, but also an honorable impact in educational policy.

This article will argue that her point of view regarding the importance of the acquisition of central process-related mathematical competencies reflects an ongoing international discourse on educational policy with regard to curricula and mathematics teaching that has been ongoing for over two decades now. We aim to show that orientation toward competencies has influenced the mathematics curricula in three different countries—Germany, Korea, and Norway—and we discuss similarities and differences with regard to how current and future challenges of students' lives are comprehended. The aim is to highlight how the teaching of competencies in mathematics education should be conceptualized and that this should not be understood as watering down mathematics, but as a shift in emphasis in how and to what depth mathematics is taught in the classroom, and how mathematics as a subject is viewed by teachers and students.

We, the team of authors, are researchers who have gotten to know each other through Gabriele's research activities and exchanges with her at the numerous congresses and conferences to which Gabriele has contributed and in which she has participated. We aim, in this contribution to her 70th birthday, to combine two heartfelt themes of Gabriele Kaiser's work – mathematical competencies and international collaboration –, and to contribute to her tireless efforts for improvements in mathematics education on a local and global (international) level.

2. What does competence orientation mean?

In order for mathematics teaching to be competence-oriented, it must first be clarified what competence or competencies mean. What makes a student competent? Many empirical studies and conceptualizations of competence in curricula are based on a concept of competence as introduced by Franz Emmanuel Weinert. He wrote an influential review of different definitions of competence in a report prepared for the OECD in the early 2000s (Weinert, 2001a, 2001b). By professional competence, Weinert means: "the cognitive abilities and skills available in or learnable by individuals to solve specific problems, and the related motivational, volitional, and social dispositions and skills to use the problem solutions successfully and responsibly in variable situations" (Weinert, 2001b, p. 27f. translated). Rychen and Salganik (2003, p. 43) add to this understanding, defining competence as "the ability to successfully meet complex demands on a particular context through the mobilization of psychosocial prerequisites (including both cognitive and non-cognitive aspects)."

Competence therefore refers primarily to knowledge and ability. Moreover, competence is understood as situation-specific, i.e., competence is context-based, and relates to cognitive components. However, the competence definition also includes affective components, such as volitional and motivational and social readiness to apply competence in various situations. It should also be noted that there is a distinction between competence as a general concept, and individual competencies (singular: competency) referring to individual content-related competence facets, such as mathematical problem solving, for instance (Weinert, 2001).

Furthermore, competencies are learnable. Therefore, the acquisition of competence in different subject areas is based on learning and experience in relevant and domain-specific situations (Pettersen

& Nortvedt, 2018, p. 951). If one tries to define competence in a subject-specific way, one is quickly confronted with the ambivalence and ambiguity of the concept of competence (Kilpatrick, 2014). Structural and content-related aspects as well as cultural traditions and educational values play an equally important role in the more precise definition of what counts as learnable mathematical competencies and what does not, and thus it must be stated that even more than twenty years after Weinert's expertise, there is no unambiguous definition of mathematical competence. However, various mathematics education research groups have presented conceptual descriptions of mathematical competence that have influenced mathematical curricula in many countries and also the development of international comparative studies in various ways (Kilpatrick, 2014; Niss et al., 2016).

One such example is the definition by Niss and Jensen (2002; 2011) and Niss (2003), who define mathematical competence as a knowledge-based resource. It is defined as follows: "mathematical competence comprises having knowledge of, understanding, doing, using and having an opinion about mathematics and mathematical activity in a variety of contexts where mathematics plays or can play a role" (p. 49, cited in Niss & Jensen, 2011). By comparison, mathematical competencies are the main constituents in mathematical competence: "The core of a competency […] is an insight-based readiness to act, where 'the action' can be both physical and behavioural – including oral – as well as mental. A valid and comprehensive assessment of a person's mathematical competencies must therefore, as a starting point, be based on identifying the presence and extent of these features in relation to the mathematical activities in which the respective person has been/is being involved." (Niss & Jensen, 2011, p. 137). Niss and Højgaard (2019) recently reviewed their framework, thereby clarifying that, in contrast to Weinert, they view competencies as purely cognitive resources without any affective facet.

Niss and Jensen's framework originated from the Danish KOM project on the further development of mathematics education and had far-reaching significance for the development of educational standards and curricula in various, mainly European, countries (Niss et al., 2016). The project distinguished eight different sub-competencies, that together constitute mathematical competence. These eight mathematical sub-competencies are: mathematical thinking competency, problem handling competency, modelling competency, reasoning competency, representations competency, symbols and formalism competency, communication competency, and aids and tools competency. Although the sub-competencies are unique and each has its own identity, they are strongly anchored to each other, and one cannot reduce one competency to the others (Niss & Jensen, 2002, p. 44).

At about the same time, a framework model for describing mathematical literacy also emerged in the United States. Kilpatrick et al.'s strand model is another well-known framework used to describe what it means to master mathematics. It is described in the National Research Council's (NRC) *Adding It Up: Helping Children Learn Mathematics* (Kilpatrick et al., 2001) and in the *RAND Mathematics Study Panel* (2003), and it specified five interwoven strands of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick et al., 2001, p. 116). This framework is based on a variety of experiences of what it means to learn mathematics, it is influenced by cognitive psychology and mathematics education. Furthermore, it recognizes knowledge, types of understanding, and skills people need in today's society (Kilpatrick et al., 2001, p. 115). Despite the differences in the definitions presented above, one finds two aspects of each definition in particular similar: being able to apply knowledge and skills, and "understanding" mathematics.

Competence orientation in mathematics education then means to organize and orient instruction through teaching and learning objectives in such a way that students can acquire competencies through appropriate learning opportunities where they gain mathematical understanding by applying mathematical knowledge and skills in varying contexts. The acquisition of competencies can be realized in a variety of teaching situations, for example, when new mathematical content is discovered by the students or developed by means of a suitable problem orientation in the classroom. The systematic practice and retention of mathematical content can also be designed in the sense of competence orientation through competence-oriented exercise formats and tasks that enable learning opportunities regarding different processes related to mathematical competence. Finally, it is important to think about competence-oriented assessment and performance measurement. Teachers need to identify the extent to which competencies have been learned in order to arrive at differentiated diagnostic judgments about possible weaknesses in students' individual competency development. In addition, teachers also need to identify what students can do for formative purposes, in order to provide feedback that can steer student learning. Therefore, teaching and assessment need to be aligned, and as such, in competence-oriented mathematics education assessment also needs to be competence oriented.

In the following, we will use analyses of the educational contexts in the three countries of Germany, Korea, and Norway (in alphabetical order), to show how these general considerations of mathematical competence have influenced educational policy in each country.

3. Competency orientation in the German educational context

In the early 2000s, as a result of the so-called "PISA shock" in Germany, sustainable educational reforms began in education policy that had competence orientation at the core of its impetus. PISA had shown that German students performed at only a mediocre level in international comparisons and that, in particular, secondary school students, i.e., students aiming for an educational qualification that would entitle them to enter general higher education, primarily mastered technical tasks with prescribed solution algorithms in mathematics, but performed worse in problem-solving and transfer tasks (Neubrand & Neubrand, 2004). In response to the only average results, which had generally been expected to be better, it was decided to introduce educational standards. Although Germany is organized on a federal level in terms of educational policy, the effort to change the educational system toward a stronger output orientation and verifiability of educational processes through monitoring and the introduction of or adherence to educational standards in all German *Länder* (states) was a nationwide political development. It was largely intended by the Standing Conference of the Ministers of Education and Cultural Affairs of the *Länder* (KMK). A first outcome was the educational standards for mathematics in 2003, which have since been applied to all *Länder* and were initially set for elementary school (grades 1–4) and the Intermediate Secondary School Certificate (after 10 years of schooling). In 2004, standards followed for the Lower Secondary School Certificate (after 9 years of schooling). Only a few years later, in 2012, standards were set for the General Secondary School Certificate (after 12 or 13 years of schooling) in mathematics. Currently, in 2021, the standards are revised and adjusted to today's educational requirements.

The conceptual basis of all standards for mathematics are the so-called basic experiences according to Heinrich Winter (1995), which emphasize the relationship of mathematics teaching to general education following the idea of *Bildung* and should be mediated to every student in the classroom. Mathematics should be understood in the following specific ways:

(1) to perceive and understand phenomena of the world around us, which concern or should concern us all, from nature, society and culture, in a specific way;

(2) mathematical objects and facts, represented in language, symbols, and formulas, as intellectual creations, as a deductively ordered world of its own kind;

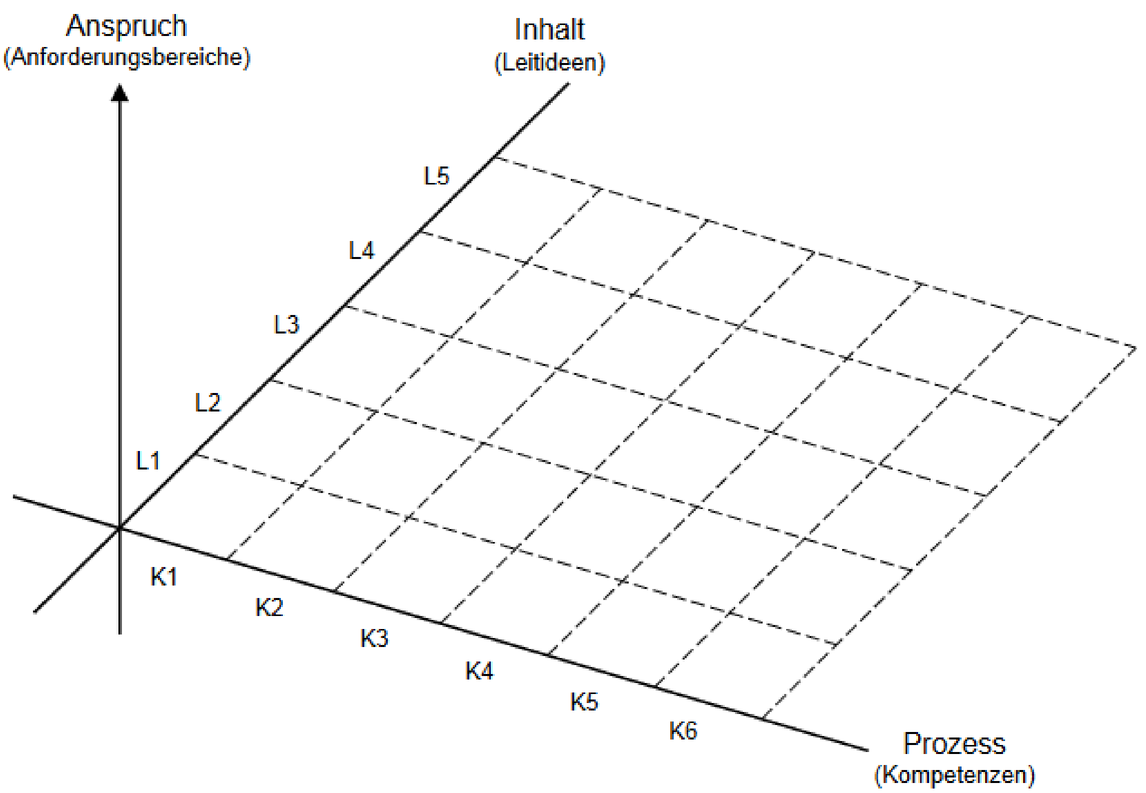
(3) to develop problem-solving skills that go beyond mathematics (heuristic skills) (Winter 1995, p.37, translated).

The standards formulated on the basis of competencies are performance standards, and for the first time shifted the subject orientation of instruction more toward mathematical processes, such as problem-solving, modelling, or mathematical argumentation, since the lack of appropriate problem-solving strategies was seen as one of the main causes of the mediocre knowledge transfer of German students identified by the PISA study.

What had been changed? One of the most significant changes that competence orientation brought to mathematics education was the formulation of the "general" mathematical competencies. As Leuders (2011) states, it had already long been the consensus in other subjects that competencies are constituted in the "functional" areas of the subject, i.e., in the communicative areas of "reading, listening, speaking, writing" in language education and not, for example, in the content areas of the subject (such as "grammar" or "regional and cultural studies"). The subject of mathematics had so far, however, defined itself exclusively in terms of content, which is why the introduction of competence standards is still discussed controversially today among mathematicians, mathematics educators, and educational stakeholders.

The core of these standards consisted of six defined general process-oriented mathematical competencies, and we here refer to the formulation chosen in the standards for the Intermediate Secondary School Certificate and the Lower Secondary School Certificate. These are: reasoning (K1), problem solving (K2), modelling (K3), using mathematical representations (K4), dealing with symbolic, formal, and technical aspects of mathematics (K5), and communicating (K6)—all of which correspond to six of the eight Danish KOM Project competencies and also take up the perspective of the NTCM standards on mathematical thinking processes, such as problem-solving or reasoning and proof. These general competencies form a process-related dimension in the competency structure model of the educational standards (depicted in Figure 1). Furthermore, five content areas—so called *Leitideen* (leading ideas)—have been formulated on the basis of which competencies can be acquired, and within which certain subject matter content is mandatory (Blum, 2015, p. 18). The *Leitideen* are: number (and algorithm) (L1), measurement (L2), space and shape (L3), functional relations (L4), and data and chance (L5) (see also Bieler, 2019). This dimension of the educational standards replaced previous content lists of the curricula, which had described the objectives of mathematics education in the *Länder* before the introduction of the educational standards. Across the different grade level standards, the wording of the guiding ideas and general mathematical competencies varies slightly, depending on different teaching focuses, but across the board, all of the standards have the same structure. In this context, content and general competencies are inseparably linked, which is symbolized by the grid in Figure 1. Werner Blum, one of the main mathematics education drivers of this educational reform (and the PhD supervisor of Gabriele Kaiser) coined the expression "There is no knitting without wool" (Blum, 2015, p. 19), which means that competencies can only be taught through concrete mathematical content. This connection is often disregarded in the criticism of competence orientation, because a content-empty competence orientation can undoubtedly be criticized, but correctly understood it always includes a strong orientation to mathematical content as well. That is why Gabriele Kaiser has pointed out, among other things, that requirements are shifting toward translation and interpretation processes when dealing with competence-oriented mathematical tasks (Kaiser & Busse, 2014). In addition to this two-dimensional conceptualization, competencies can be formulated on different requirement levels that correspond to the cognitive demands of mathematical activities, which shapes a third dimension when locating competencies for planning and justification purposes. Niss et al. (2017) therefore summarize in their recent review of the implementation of competence orientation in different countries that in Germany the general mathematical competencies "are put into a three-dimensional structure, with five mathematical content areas ('Numbers'; 'Measurement'; 'Space and Shape'; 'Functional Dependencies'; 'Data and Chance') and three levels of mastery as in the early PISA frameworks ('Reproduce'; 'Make Connections'; 'Generalize and Reflect') as the other two dimensions" (Niss et al. , 2017, p. 245).

Fig. 1: The three-dimensional conceptualization of the mathematics educational standards in Germany (Roppelt et al., 2013, p. 25).

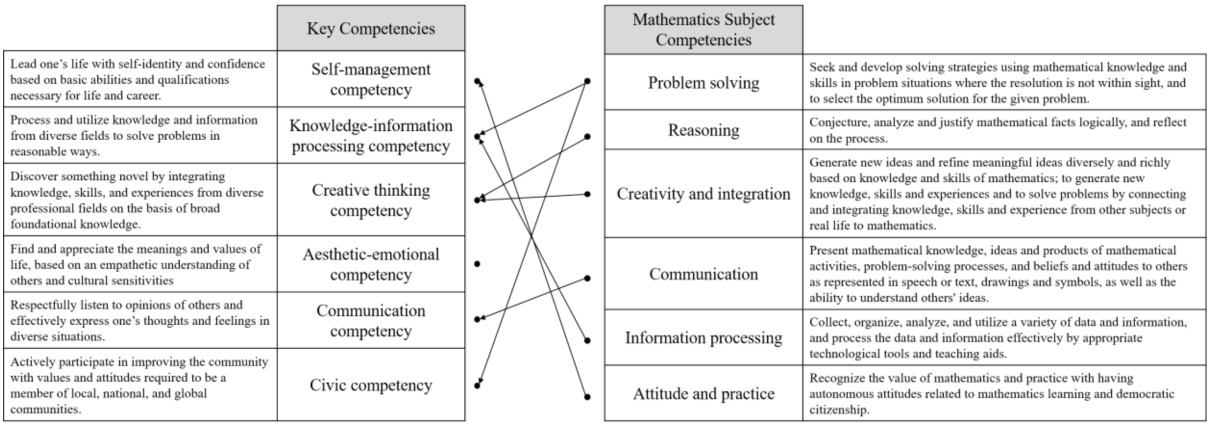


4. Competency-based curriculum in the Korean educational context

The Korean national curriculum documents, which are revised once every six years on average, regulate the content and the scope of education in the school curriculum. Korean national curriculum documents are divided into documents on general guidelines and documents on subject curriculum. General guidelines reflect the vision for future societies and the needs of the nation and society. General guidelines also address revisions of the overall goals and vision for an educated person in the curriculum, along with guidelines for teaching, learning, and assessment. The mathematics subject curriculum especially addresses revision in the characteristics and goals of mathematics, the mathematics courses, the inclusion and exclusion of mathematical content, guidelines for mathematics teaching and learning, and assessments, among other topics.

The 2015 revised curriculum that is currently implemented in Korea reflects national and social needs represented by nurturing creative and integrative learners. This curriculum explicitly introduces the concept of key competencies for the first time. At the time of the 2015 revised curriculum, the perspective on education in Korea was focused on developing competencies in the process of learning knowledge, rather than on the process of delivering knowledge. In order to realize this perspective on education in the curriculum, the concept of key competencies is emphasized in the 2015 national revised curriculum. In the general guidelines of the 2015 revised curriculum, six key competencies are presented: self-management competency, knowledge- and information-processing competency, creative thinking competency, aesthetic emotional competency, communication competency, and civic competency (Korean Ministry of Education, 2015a). Subject competencies were reinterpreted from key competencies, considering the contents and teaching of each subject. Six mathematics subject competencies were suggested: problem solving, reasoning, creativity and integration, communication, information processing, and attitude and practice (Korean Ministry of Education, 2015b). The relationship between key competencies and mathematics subject competencies is shown in Figure 2.

Fig.2: The Relationship between Key Competencies and Mathematics Subject Competencies



Sub-attributes of key competencies and mathematics subject competencies are described in the 2015 revised curriculum documents to specify the concept of each competency. Based on the sub-attributes, the relationship between key competencies and mathematics subject competencies can be connected. This relationship is not a one-to-one correspondence. One mathematics subject competency can be linked with two key competencies. By contrast, there is one key competency that is not connected to any of the mathematics subject competencies. Knowledge information processing competency and creative thinking competency are reified as problem solving, reasoning, creativity and integration, and attitude and practice competencies. Other key competencies are also related to the mathematics subject competencies. The self-management competency is connected with attitude in the learning process of mathematics. Civic competency is connected to the collaborative problem solving that is described in the problem-solving competency. Communication competency is connected to mathematical communication based on text, drawing, and symbols as well as the ability to understand others. Unlike other key competencies, aesthetic-emotional competency is not emphasized in any mathematics subject competency.

Korean mathematics subject competencies were closely related to mathematical competencies in the international trends of competency-based curriculum. The concepts of problem-solving competency, reasoning competency, and communication competency are coherent with mathematical process standards in NCTM (2000). The mathematical process of the 2009 revised mathematics curriculum refers to a skill or ability that must be activated when connecting various phenomena around students with mathematics, and when solving problems occurs in various situations (Shin et al., 2011). The mathematical process consists of three components—problem solving, reasoning, and communication—and the three components were selected as mathematics subject competencies in the 2015 revised curriculum. NCTM (2000) emphasizes students' connections of mathematical ideas that can be realized from recognizing and understanding the interconnections between mathematical ideas, and recognizing and applying mathematics in nonmathematical contexts. The meaning of creativity and integration competency will serve students well as they pursue the connecting and integration of mathematical knowledge to problem solving in other subjects and real-world problems. The term “information processing” competency means the ability to collect, organize, analyze, and utilize data and to use technological tools. While it is good to avoid excessive formalization and symbolization of mathematics, the trends of the international mathematics curriculum emphasizing the use of technological tools have been reflected (e.g., CCSSI, 2014). Attitude and practice competency, unlike other competencies, reflect the affective domains that encourage students to study mathematics. The results of international comparison studies of academic achievement, such as PISA and TIMSS, show that Korean students attained high achievement scores but low levels in the affective domains, such as motivation, confidence, and enjoyment.

In the process of developing the Korean mathematics curriculum, discussions were conducted on the inclusion and exclusion of mathematical content in order to reduce the burden of students’ learning (e.g., Yoon et al., 2021). In the 2015 revised mathematics curriculum, the meaning of creativity and integration competency, and attitude and practice competency, are ambiguous for teachers, making teaching and assessing competencies difficult. The 2022 revised curriculum develops a competency-based curriculum that aligns with the 2015 revised curriculum. The 2022 revised mathematics curriculum is expected to constitute mathematics subject competencies, elaborating upon the definitions and categories of competencies.

5. Competency-based mathematics curricula in Norway

In Norway, education at the primary and secondary levels is regulated by the Education Act and the national curriculum (Lovdata, 1998; National Directorate for Education and Training [NDET], n.d., a). The main aim of the Norwegian educational system is to “open doors to the world and give the pupils and apprentices historical and cultural insight and anchorage” (NDET, n.d., a, p. 1). Values such as equity, equality, and cultural understanding are highlighted in the overarching introduction to the curriculum (Nortvedt, 2018). Moreover, and in line with Niss (1996), the goal of mathematics teaching is to ensure that all students develop knowledge, skills, and attitudes by which to master their personal lives, take part in work life, and contribute to society. Accordingly, critical thinking is highlighted as a key skill both in the overarching introduction and in the mathematics curriculum.

The Education Act identifies inclusion, adaptation, and assessment for learning as steering principles for achieving these goals (Lovdata, 1998).

Traditionally, the humanistic idea of *Bildung* (Jahnke, 2019) has been seen as a primary goal of compulsory education in Norway. This is visible, for instance, in the curriculum that was introduced in 1997 that emphasized the principle of educating “the whole person.” This has been carried forward to the most recent curriculum, implemented in 2020, in which the goals of education include opportunities to form a mathematical identity (NDET, n.d., a).

Norway got its first national curriculum in 1890. Since then, the curriculum has been revised several times, with the latest revisions introduced in 1987, 1997, 2006 [2013], and 2020. In each revision, changes have been made to the mathematics curriculum to enhance the level of mathematical literacy in the general public. Since 2006, the mathematics curriculum has been influenced by the notion of mathematical competence (Niss et al., 2016), and in particular by Niss and Jensen (2002) and Kilpatrick et al. (2001). The emphasis on mathematical knowledge has gradually been weakened. In 1987, problem solving was identified as one of the main strands, alongside traditional strands such as numbers, algebra, geometry, and functions. At this point, problem solving strategies were seen as part of students' mathematical knowledge and not connected to competence. In 1997, problem solving was no longer a strand, but rather was identified as a main teaching method by which to ensure that students developed mathematical understanding rather than merely computational skills.

With the 1997 curriculum, the focus shifted from product (knowledge) to process-oriented mathematics education. This can be viewed as a starting point for the focus on mathematical competence. With the 2006 curriculum, competency goals in mathematics were introduced. In addition, more attention was paid to literacy, identifying reading, writing, and the communicating of mathematics as basic skills and core aspects of mathematical learning alongside digital skills (NDET, 2017). This curriculum was revised in 2013 to make the learning progression in, for instance, reading mathematics, clearer. In the 2020 curriculum, the competency goals are carried forward. At the same time, the curriculum was organized around so-called core elements, two of which were modelling and problem solving (Berget & Bolstad, 2019), while at the same time carrying forward the basic skills and introducing two interdisciplinary topics to the mathematics curriculum for compulsory education: health and life skills, and democracy and citizenship.

The Norwegian mathematics curriculum is developed in a combined top-down and bottom-up process: while the Ministry of Education develops general guidelines and a mandate, the process is regulated by the Directorate for Education and Training; the curriculum content, however, is developed by a team of appointed teachers, mathematicians, and mathematics teacher educators. Their proposed curriculum is revised after national hearings where all teachers have the opportunity to respond to the proposed core elements and competency goals. This combined approach allows mathematics teachers across the country to engage in curriculum development.

Developing the 2020 curriculum was a three-stage process. First, national white papers describing the goals of future educational papers were developed, identifying in-depth learning as the steering principle (Norwegian Ministry of Education, 2014; 2015). For the mathematics curriculum, this meant leaving the spiral principle, where students would learn a little more about, for instance, numbers, each year, to organizing the curriculum around big ideas or block teaching, such as, for instance, teaching all competency goals connected to functions in the same school year. Next, the core elements were developed by a national committee, identifying five core elements in addition to a sixth element comprising the traditional mathematical strands: exploration and problem solving, modelling and applications, reasoning and argumentation, representation and communication, abstraction and generalization, and mathematical fields of knowledge (see Table 1). Finally, the curriculum group used the white papers, the core element descriptions, and the national basic skills framework to develop the national mathematics curriculum for compulsory education (grades 1–10) (NDET, n.d., b).

Table 1 presents the basic skills and core elements in the Norwegian mathematics curriculum for compulsory education. A common denominator of the skills and elements is the emphasis of development: from using informal, everyday language-oriented representations, forms of language, and communication, to using formal, symbolic representations, arguments, and ways of communication. This is evident, for instance, in the description of oral skills and in the core element "abstraction and argumentation."

Moreover, the curriculum emphasizes programming and computational thinking. For instance, the "exploring and solving problems" core element includes computational thinking as a tool with which to develop strategies and procedures for solving problems. Students should assess whether problems would be best solved with or without the use of digital tools.

The relationships between the basic skills and the core elements first become clear when linked to the competency goals. In grade 10, for instance, the first competency goal states that students should be able to explore and generalize multiplication of polynomials both algebraically and geometrically. This goal is linked to the basic skills of numeracy and writing, indicating that this goal can be utilized to solve applied problems, and that by working toward this competency goal, students develop their basic mathematical writing skills. The goal is also linked to the core elements "exploration and problem solving" and "abstraction and generalization." Applications and solving problems are embedded in both the basic skill of numeracy and in the core element "exploration and problem solving." Likewise, mathematical language is at the core of both the basic skill of writing and of the core element "abstraction and generalization." Linking basic skills and core elements to competency goals, the curriculum group signals how teachers might interpret the short goal descriptions in the mathematics curriculum and what to focus on when planning teaching and learning activities. When only a few of the competency goals are connected to interdisciplinary topics, for instance, "plan, carry out, and present an exploratory work related to one's personal finances" (grade 10), the curriculum group at the same time signals that these are less prominent in reaching the goals of mathematics teaching in Norway—perhaps in contrast to the stated goals in the overarching part and in the introduction to the mathematics curriculum.

In alignment with the competency-oriented curriculum, the national mathematics exam also emphasizes mathematical competence demanding students to solve applied known and unknown problems. Solving includes reasoning, evaluating and communicating mathematical ideas and solution procedures. In addition to problem solving, students need to demonstrate that they can explore and model problems (https://www.udir.no/eksamen-og-prover/eksamen/eksempeloppgaver/eksempeloppgaver-i-matematikk-grunnskolen/). As such, students need to demonstrate mastery both of basic skills and of core elements. Moreover, each exam task is designed to assess students' competence in accordance with at least one competency goal in the curriculum. Taken together, this indicates that the main emphasis is on assessing students' mathematical competence.

Tab. 1: Basic skills and core elements in the Norwegian mathematics curriculum

Basic skills in mathematics	
Creating meaning by talking in and about mathematics, communicating ideas, and discussing mathematical problems, strategies, and solutions with others.	Oral skills
Describing and explaining relationships, findings, and ideas using well-reasoned	Writing

Core elements	
Exploration and problem solving	Searching for patterns and relationships; emphasizing strategies and procedures; developing methods for solving unfamiliar problems.
Modelling and applications	Gaining insight into how mathematical

representations, including solutions to problems.	
Creating meaning in texts from everyday life, society, and the field of mathematics; being able to sort information, analyze and assess its form and content, and summarize information in multimedia texts.	Reading
Using mathematical representations, concepts, and methods to make calculations and assess whether solutions are valid; recognizing concrete problems that can be solved using numeracy skills and formulating questions about these.	Numeracy

	models are used to describe everyday lives, working life, and society in general; creating models and critically assessing whether models are valid and identifying limitations.
Reasoning and argumentation	Following, assessing, and understanding mathematical trains of thought; formulating one's own reasoning to understanding and solving problems; proving the validity of arguments.
Representation and communication	Expressing mathematical concepts, relationships, and problems using concrete, contextual, visual, verbal, and symbolic representations; communicating mathematically using both everyday and mathematical language.

Using graphing tools, spreadsheets, CAS, dynamic geometry software, and programming to explore and solve mathematical problems including finding, analyzing, processing, and presenting information using digital tools.	Digital skills

Abstraction and generalization	Formalizing thoughts, strategies, and mathematical language; moving away from using concrete descriptions to more formal symbolic language and reasoning.
Mathematical fields of knowledge	The mathematical fields of knowledge include numbers and understanding numbers, algebra, functions, geometry, statistics, and probability.

Conclusion

In the synopsis of the three different approaches of the various countries with regard to the orientation of curricula along general and mathematical competencies, it is noticeable that the acquisition of mathematical knowledge is no longer defined and legitimized by content alone, which, especially in Germany, led to massive changes. Rather, mathematical competencies are seen in the context of general abilities related to everyday life, which can also be transferred to different situations or to other subjects. Also, mathematical knowledge that is to be learned is seen in the context of various mathematical activities and processes that reflect the way mathematicians work, such as reasoning and justifying or problem solving, and which, however, vary in the individual countries according to specific educational policy emphases and traditions. Important influencing factors in all countries are the programmatic preparatory work of the NCTM or the Danish KOM project, without which the effort to introduce educational standards would certainly not have been as extensive and similar.

It is also striking that in all countries a strong link is seen between mathematical competence and the development of critically mature citizens—often also linked to a humanistic educational ideal or the concept of *Allgemeinbildung* or *allmendannelse* (Biehler, 2019). Examples of this are the inclusion of positive attitudes toward mathematics in the canon of competencies in Korea and the inclusion of interdisciplinary topics in Norway. Basically, what all countries have in common is that the standards for teaching mathematics relate to preparing students for the future, whether in economic, democratic, or scientific terms. The Covid-19 pandemic and increasing climate change are examples of the challenges students will also have to deal with in the future and the extent to which, for example, critical reading and understanding of data is a crucial mathematical skill.

However, the transition to a competency-based approach in education is not an easy one. Implementation through state-mandated curricula does not equal successful implementation of educational reforms at the level of classroom teaching. Thus, competence orientation or the development toward competence-oriented curricula is much discussed in many countries. However, this may not be a problem of educational standards, but of educational reforms in general. This presents challenges, but also opportunities.

One challenge is certainly that the introduction of competencies changes not only teaching, but that central school-leaving exams have to be redesigned as well. The challenges of progressive digitization also lead to a continuous adaptation of competency standards (Geraniou & Jankvist, 2019), for example, when it comes to how competencies can be acquired with digital tools or how competencies can be taught in online lessons (here, there is all too often the danger that teachers fall back into a content-oriented mathematics teaching and the mathematical activity of students is neglected).

The influence that mathematics education research can have on educational reforms is, of course, limited, as is the case with many political decisions. However, educational research through international large-scale studies, such as TIMSS or PISA—and thus also mathematics educational research—repeatedly contributes to new findings on students' mathematical competencies that can be used in educational policy. For example, Nortvedt (2018, p. 440) describes that understanding students' mathematical thinking and experiences with mathematics teaching and learning has the potential to contribute to an education for all and to equity and equality in schools, including equal opportunities for majority and immigrant students. Mainly, teacher training and professional development are the main sources of influence for mathematics education researchers. However, the direct influence of experts who scientifically accompany the creation of educational reforms, such as the introduction of standards or the development of final examinations, and who provide feedback on the creation of curricula, as in the example of Norway, must also be considered important. Mathematics education researchers have experience from many different countries and educational contexts. To this end, it is essential that mathematics education researchers network internationally and have an overview of the developments in mathematics teaching in other countries and where they can learn from each other. Gabriele Kaiser has exemplified this in her long academic career, for example through her research stays in England (Kaiser, 1999) or in recent years in Australia.

We want to continue this legacy productively and will continue to follow and shape the development of competence orientation in the countries together in the future.

References

Berget, I. K. L., & Bolstad, O. H. (2019). Perspektiv på matematisk modellering i Kunnskapsløftet og Fagfornyinga [Perspectives on mathematical modelling in the curriculum reforms in 2006 and 2020]. *Nordisk Tidsskrift for Utdanning Og Praksis*, *13*(1), 83-97. https://doi.org/10.23865/up.v13.1882

Biehler, R. (2019). Allgemeinbildung, Mathematical Literacy, and Competence Orientation. In H.N. Jahnke & L. Hefendehl-Hebeker (Eds.), *Traditions in German-Speaking Mathematics Education Research* (pp. 141–171). Springer.

Blum, W. (2015). Zur Konzeption der Bildungsstandards Mathematik für die Allgemeine Hochschulreife. In W. Blum, S. Vogel, C. Drüke-Noe & A. Roppelt (Eds.), *Bildungsstandards aktuell: Mathematik in der Sekundarstufe II* (pp. 16–30). Bildungshaus Schulbuchverlage.

CCSSI (2014). Common Core State Standards for Mathematics. Retrieved from http://www.corestandards.org/math

Geraniou, E., & Jankvist, U. T. (2019). Towards a definition of "mathematical digital competency." *Educational Studies in Mathematics, 102*(1), 29-45.

Jahnke, H. N. (2019). Mathematics and Bildung 1810 to 1850. In Jahnke H., Hefendehl-Hebeker L. (eds) Traditions in German-Speaking Mathematics Education Research. ICME-13 Monographs. Springer. https://doi.org/10.1007/978-3-030-11069-7_5.

Kaiser, G. (1999). *Unterrichtswirklichkeit in England und Deutschland. Vergleichende Untersuchungen am Beispiel des Mathematikunterrichts*. Beltz.

Kaiser, G. & Busse, A. (2014). Hamburger Mathematikabitur im Kreuzfeuer der Kritik. *Mitteilungen der GDM, 97*, 28–31.

Kilpatrick, J. (2014). Competency Frameworks in Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 110–113). Springer.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). Adding it up: Helping children learn mathematics. The National Academies Press. https://doi.org/10.17226/9822

Korean Ministry of Education (2015a). *National guidelines for the elementary and secondary curriculum*. Proclamation of the Ministry of Education #2015-74[Annex 1]. Seoul: Author.

Korean Ministry of Education (2015b). *Mathematics curriculum*. Seoul: Author.

Leuders, T. (2011). Kompetenzorientierung – eine Chance für die Weiterentwicklung des Mathematikunterrichts? In K. Eilerts, A.H. Hilligus, G. Kaiser & P. Bender (Eds.) *Kompetenzorientierung in Schule und Lehrerbildung. Perspektiven der bildungspolitischen Diskussion, der empirischen Bildungsforschung und der Mathematik-Didaktik. Paderborner Beiträge zur Unterrichtsforschung und Lehrerbildung*, Volume 15. LIT Verlag.

Lovdata (1998). Act relating to primary and secondary education and training (the Education Act). Retrieved September 6, 2021 from https://lovdata.no/dokument/NLE/lov/1998-07-17-61.

NCTM (2000). *Principles and Standards for School Mathematics*. Reston, VA: The National Council of Teachers of Mathematics.

Niss, M. (2003). The Danish KOM project and possible consequences for teacher education. In R. Strässer, G. Brandell, B. Grevholm, & O. Helenius (Eds.) (2003). Educating for the future. Proceedings of an International Symposium on Mathematics Teacher Education (pp. 178–192). Gothenburg: Royal Swedish Academy of Science.

Niss, M. (1996). Goals of mathematics teaching. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), International handbook on mathematics education (pp. 11–47). Springer Netherlands.

Niss, M. & Højgaard, T. (2019) Mathematical competencies revisited. Educational Studies in Mathematics, 102, 9–28.

Niss, M., & Jensen, T. H. (Eds.). (2002). Kompetencer og matematiklæring—Idéer og inspiration til udvikling af matematikundervisning i Danmark. Uddannelsesstyrelsens temahæfteserie nr. 18. Copenhagen: The Ministry of Education.

Niss, N., & Jensen, T. H. (Eds.). (2011). Competencies and Mathematical Learning. Ideas and inspiration for the development of mathematics teaching and learning in Denmark. English translation of parts I–VI of Niss and Jensen (2002). Tekster nr. 485. Roskilde University: IMFUFA. https://pure.au.dk/ws/files/41669781/THJ11_M

Niss, M., Bruder, R., Planas, N. et al. (2016). Survey team on: conceptualisation of the role of competencies, knowing and knowledge in mathematics education research. *ZDM Mathematics Education*, *48*, 611–632. https://doi.org/10.1007/s11858-016-0799-3

Niss, M., Bruder, R., Planas, N., Turner, R. & Villa-Ochoa, J.A. (2017). Conceptualisation of the Role of Competencies, Knowing and Knowledge in Mathematics Education Research. In G. Kaiser (Ed.), *Proceedings of the 13th International Congress on Mathematical Education, ICME-13 Monographs* (pp. 235–248). Springer.

Neubrand, J., & Neubrand, M. (2004). Innere Strukturen mathematischer Leistung im PISA-2000-Test. In M. Neubrand (Ed.), *Mathematische Kompetenzen von Schülerinnen und Schülern in Deutschland. Vertiefende Analysen im Rahmen von PISA 2000* (pp. 87–108). VS Verlag für Sozialwissenschaften.

Nortvedt, G. (2018). Policy impact of PISA on mathematics education: the case of Norway. Eur J Psychol Educ (2018) 33:427–444. https://doi.org/10.1007/s10212-018-0378-9

Norwegian National Directorate for Education and Training (NDET) (n.d., a). Læreplanverket [National curriculum]. Udir.no. https://www.udir.no/laring-og-trivsel/lareplanverket/

Norwegian National Directorate for Education and Training (NDET) (n.d., b). Curriculum for Mathematics Year 1–10 (MAT01-05). Udir.no. https://www.udir.no/lk20/mat01-05?lang=eng.

Norwegian National Directorate for Education and Training (NDET) (2017). Framework for basic skills. https://www.udir.no/laring-og-trivsel/rammeverk/rammeverk-for-grunnleggende-ferdigheter/,

Norwegian Ministry of Education (2014). NOU 2014:7. Elevenes læring i fremtidens skole — Et kunnskapsgrunnlag [Student learning in the future school]. https://www.regjeringen.no/no/dokumenter/NOU-2014-7/id766593/

Norwegian Ministry of Education (2015). NOU 2015:8. Fremtidens skole — Fornyelse av fag og kompetanser [The school of the future—revising subjects and competencies]. https://www.regjeringen.no/no/dokumenter/nou-2015-8/id2417001/

Pettersen, A., & Nortvedt, G.A. (2018). Identifying Competency Demands in Mathematical Tasks: Recognising What Matters. *International Journal of Science and Mathematics Education*, *16*, 949–965.

RAND Mathematics Study Panel. (2003). Mathematical proficiency for all students. Toward a strategic research and development program in mathematics education. Santa Monica, CA: RAND.

Roppelt, A., Blum, W., & Pöhlmann, C. (2013). Beschreibung der untersuchten mathematischen Kompetenzen. In H.A. Pant, P. Stanat, U. Schroeders, A. Roppelt, T.Siegle & C. Pöhlmann (Eds.), *IQB Ländervergleich 2012. Mathematische und naturwissenschaftliche Kompetenzen am Ende der Sekundarstufe I* (pp. 23–37). Waxmann.

Rychen, D.S., & Salganik, L.H. (2003). *Key competencies: For a successful life and a well-functioning society*. Hogefe and Huber.

Shin, I. S., Kwon, K. S., Jang, M. S., Hwang, H. J., Kim, D. W., Lee, D. H., ... , Song, H. J. (2011). *A study on mathematics curriculum according to 2009 revised curriculum.* Korea Foundation for the Advancement of Science & Creativity.

Yoon, H., Bae, Y., Lim, W., & Kwon, O. N. (2021). A story of the national calculus curriculum: how culture, research, and policy compete and compromise in shaping the calculus curriculum in South Korea. *ZDM–Mathematics Education*, *53*(3), 663–677.

Weinert, F. E. (2001a). Concept of competence: a conceptual clarification. In D.S. Rychen & L.H. Salganik (Eds.), *Defining and selecting key competencies* (pp. 45–65). Hogrefe & Huber.

Weinert, F. E. (2001b). Vergleichende Leistungsmessung in Schulen – eine umstrittene Selbst-Verständlichkeit. In F.E. Weinert (Ed.), *Leistungsmessungen in Schulen* (pp. 17–31). Beltz.

Winter, H. (1995). Mathematikunterricht und Allgemeinbildung. *Mitteilungen der Gesellschaft für Didaktik der Mathematik, 61,* 37–46.