

A new joint optimization method for design and operation of multi-reservoir system considering the conditional value-at-risk

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Abstract:

The joint design and operation of multi-reservoir systems is a vital issue for reservoir management. Existing studies mostly focus on determining the optimal scheme by establishing an optimization model and relying on intelligent algorithms. However, the research on the mutual feedback mechanism between the flood control capacity of each reservoir has not been adequately addressed. The overall aim of this paper is to propose a new joint optimization method for design and operation of multi-reservoir systems considering the conditional value-at-risk ($CVaR_{\alpha}$). In the proposed method, the flood damage assessment with $CVaR_{\alpha}$ for a multi-reservoir system is constructed firstly, and then the feasible flood storage combination scheme (FSCS) of reservoirs in the system is deduced. Finally, the tradeoffs between the hydropower generation benefit and flood damage loss have been analyzed. Selecting China's Ankang-Danjiangkou Cascade Reservoirs as a case study, the results indicate that (1) the $CVaR_{\alpha}$ value is more sensitive to changes in the flood storage of Danjiangkou reservoir (larger and downstream) than that of Ankang reservoir (smaller and upstream); (2) the feasible area of the FSCSs can be described as a triangle, and the boundary of this feasible interval is determined by the allowable minimum flood storage (AMFS) value of each reservoir; and (3) the relationship between the hydropower generation benefit and the flood damage assessment index $CVaR_{\alpha}$ is not monotonic. These findings are helpful for understanding the relationship between flood storage values of each reservoir from the perspective of the entire multi-reservoir system.

Keywords: Joint design operation; Multi-reservoir system; Flood storage; Conditional value-at-risk; Flood damage assessment.

1. Introduction

The joint design and operation of multi-reservoir systems is a necessary technical practice to realize the safe and efficient utilization of flood resources (Dogan Mustafa et al., 2021; Ming et al., 2021; Yeh, 1985; Yun and Singh, 2008; Zhou et al., 2018) when facing the increase in the dimension of the coordinated flood control reservoirs (Cheng et al., 2021; He et al., 2019). Specifically, the flood limited water level (FLWL), which corresponds to the reservoir flood storage value, is a vital parameter for controlling the trade-offs between operation objectives of flood control and water conservation (Jiang et al., 2015; Liu et al., 2015a). In China, the FLWL is usually determined by flood routing of design floods under the condition that the flood prevention risk does not increase (Liu et al., 2015b; Zhang et al., 2018). However, the traditional design method for the flood storage is separately derived from the perspective of each individual reservoir, and is not intended to maximize the benefits for the entire multi-reservoir system. Therefore, the joint design method of deriving optimal flood storage combination scheme (FSCS) for multi-reservoir systems should be developed.

Many researchers have already focused on the joint optimal design of the flood storage values for the complex multi-reservoir systems (Gong et al., 2020; Tan et al., 2017). Chen et al. (2012) deduced the feasible interval of the reservoir flood storage in a multi-reservoir system by establishing a composition and decomposition-based model, which consists of an aggregation module, a storage decomposition module and a simulation operation module. Zhou et al. (2014) described a joint operation and dynamic control model of FLWL for a mixed reservoir system, and improved the hydropower benefit and the water resource utilization without reducing the originally designed flood prevention standards. Hui and Lund (2015) proposed a flood storage allocation model for parallel reservoirs with the objective of minimizing the downstream damage caused by the possible high peak flow of the upper stream reservoirs. Ouyang et al. (2015) presented a multi-objective optimal design model for FLWLs of multi-reservoir system, which can make better use of water resources without increasing the flood control risk. However, previous studies mostly focus on determining the optimal scheme by establishing an optimization model and relying on intelligent algorithms, which are lack of identifying the mutual feedback mechanism between the flood control capacity of each reservoir.

Recently, the response relationship between the individual reservoir's flood storage and its benefit (which mainly refers to hydropower generation and water supply) has been studied by a large number of researchers, and the law of feedback mechanism can be summarized as follows (Dogan et al., 2021): the hydropower generation or water supply benefit of the single reservoir system shows a monotonous correlation with the reservoir flood storage value, that is, the larger the flood storage value is, the greater the flood control capacity (the lesser the hydropower generation or water supply benefit) is. However, the above regular pattern is not applicable to the multi-reservoir system because of the complex hydrological and hydraulic connection between the reservoirs in the system. The impact on the overall benefit of the multi-reservoir system caused by one of reservoirs' flood storage is uncertain, and then the maximization of the benefit for the multi-reservoir system cannot be reached by only re-designing the flood storage of a single reservoir in the system. Therefore, it is necessary to analyze the mutual feedback mechanism between reservoirs and their contribution to benefits from a system perspective, which motivated this study. This paper is dedicated to propose a new joint optimization method for design and operation of multi-reservoir system considering the conditional value-at-risk ($CVaR_\alpha$).

$CVaR_\alpha$, a classic finance index of economic analysis, is defined as the expected value of all losses above the selected probability, which can reflect the probability and magnitude of damage loss simultaneously. In fact, $CVaR_\alpha$ has already been commonly applied to water resources modeling (Yazdi et al., 2016), and it mainly focuses on water allocation policy (Bakhtiari et al., 2020; Li et al., 2018; Shao et al., 2011; Yamout et al., 2007), distribution of water rights (Hu et al., 2016; Zhang et al., 2020), and risk analysis on the multi-objective water resources optimization problems (Khorshidi et al., 2019; Naserizade et al., 2018; Wang et al., 2017; Zhang and Guo, 2018). Webby et al. (2006) optimized multi-objective problem for amenity and environment flows against flood risk with the help of $CVaR_\alpha$ index. Piantadosi et al. (2008) determined a policy for management of urban storm-water by combining stochastic dynamic programming with $CVaR_\alpha$. Soltani et al. (2015) established an objective function based on $CVaR_\alpha$ for planning agricultural water and return flow allocation in river systems. Fu et al. (2018) used a conditional value-at-risk two-stage stochastic programming model to plan regional water allocations. Ermoliev et al. (2019) built a stochastic optimization model for risk-based reservoir management by considering $CVaR_\alpha$ as a constraint. Furthermore, Zhang et al. (2018) incorporated the concept of $CVaR_\alpha$ into reservoir flood damage loss evaluation and proved it is reliable to take account of the $CVaR_\alpha$ value as a constraint when compared with the traditional risk-based approach, which is the method basis of this study.

The primary purpose of this paper is to propose a new optimal method for multi-reservoir systems and identify the mutual feedback mechanism between the flood storage of each reservoir. The specific objective is to extend the establishment of the flood damage assessment index $CVaR_\alpha$ from a single reservoir system to a multi-reservoir system, and to deduce the possible feasible FSCSs for a multi-reservoir system. Then, an optimization model which aims to maximize the hydropower generation benefit of the multi-reservoir system is built to find the optimal reservoir flood storage combination scheme.

The remainder of this paper is organized as follows. Section 2 describes the flood damage assessment with $CVaR_\alpha$ for a single reservoir system and a multi-reservoir system, and the establishment of the optimization model. A case study of Ankang-

Danjiangkou reservoir system is addressed in Section 3. Then, results and discussions are shown in Section 4. Finally, conclusions are given in Section 5.

2. Methodology

The framework of the proposed joint optimization method for design and operation of multi-reservoir system considering the conditional value-at-risk is shown in Fig. 1, and the processes are summarized below:

(1) The $CVaR_{\alpha}$ index for the multi-reservoir system is established in the flood damage assessment module, which reflects its relationship with FSCS and the magnitude of design floods (Section 2.2).

(2) An optimal design FSCS model is proposed with the objective of maximizing the hydropower generation benefits in the multi-reservoir system without increasing the flood damage loss, the hydropower generation module of which is used to build the relation between the objective function and FSCS (Section 2.3).

[Please insert Figure 1 here]

2.1 Flood damage assessment with $CVaR_{\alpha}$ for a single-reservoir system

Value-at-risk (VaR_{α}) represents the maximum loss with a given confidence level α over a specified time horizon, which is derived by using cumulative probability distribution function of a random variable. $CVaR_{\alpha}$, a modified form of VaR_{α}, is defined as the expected loss given that the loss exceeds VaR_{α}. Let $L(x,\theta)$ be a loss function of a decision vector x and a stochastic vector θ. The $CVaR_\alpha$ at a given confidence level $\alpha \in [0, 1]$ can be defined as follows (Rockafellar and Uryasev, 2002):

$$\begin{aligned} CVaR_\alpha &= E\left[L(x,\theta) \mid \varphi(x,\theta) \geq \alpha\right] \\ &= \frac{\int_{F_\alpha}^{F_{\max}} L(x,\theta) f\left[L(x,\theta)\right] dL}{1-\alpha} \end{aligned} \quad (1)$$

where E denotes the expected value operator, F_α is the VaR_α value corresponding to a given confidence level α ($VaR_\alpha = \min\left[L(x,\theta) \mid \varphi(x,\theta) \geq \alpha\right]$), $F_{\max}$ is the maximum value of loss function, $f(\cdot)$ is the probability distribution function for flood damage, $L(x,\theta)$ is a continuous or discrete loss function, and $\varphi(x,\theta)$ is the cumulative distribution function of loss. The units of variables $CVaR_\alpha$, VaR_α, F_α, $F_{\max}$, $L(\cdot)$ determined by the actual application problem are the same, while the units of variables $f(\cdot)$, $\varphi(\cdot)$, α are dimensionless. It should be noted that the relationship between the confidence level α and the flood risk probability of the reservoir system R is $\alpha+R=1$ (Detailed demonstration has been presented by Zhang et al. (2018)).

The reservoir flood damage assessment in this study is established by considering the reservoir downstream floodplain. $L(x,\theta)$ is a loss function for the reservoir flood storage value (m^3) (or the FLWL value (m)) x and the reservoir inflow process θ corresponding to design frequency $p(\theta)$ (%), respectively.

$$L(x,\theta) = c \cdot w_f(x,\theta) \quad (2)$$

where $w_f(\cdot)$ (m^3) represents the flood volume which needs to be diverted into the downstream floodplain, and c (yuan/m^3) is the unit cost for $w_f(\cdot)$. It should be noted that the unit cost of loss function is assumed as a constant, which is out of the scope of this study. The detailed description about the formulation of w_f for a single reservoir is provided in Supplement B Section by Zhang et al. (2018).

2.2 Flood damage assessment with $CVaR_\alpha$ for a multi-reservoir system

The flood damage assessment for the multi-reservoir system is built as Equation 3, which is extended from the single-reservoir system's flood damage assessment.

$$CVaR_{k,\alpha} = \frac{\int_{F_{k,\alpha}}^{F_{k,\max}} L_k\left(x_1, x_2, \ldots, x_n, \theta_k\right) f_k\left[L_k\left(x_1, x_2, \ldots, x_n, \theta_k\right)\right] dL_k}{1-\alpha} \tag{3}$$

where n is the number of upstream reservoirs corresponding to the downstream flood control point k, x_i represents the flood storage (m^3) (or the FLWL (m)) of the ith reservoir, $x_1, x_2, \ldots, x_n$ represents the flood storage combination scheme, θ_k is the multi-reservoir system's inflow process corresponding to design frequency $p\left(\theta_k\right)$ (%), $L_k\left(\cdot\right)$ is the loss function of the flood control point k, $F_{k,\alpha}$ is the $VaR_{k,\alpha}$ value corresponding to a given confidence level α, $F_{k,\max}$ is the maximum value of loss function $L_k\left(\cdot\right)$, and $f_k\left(\cdot\right)$ is the probability distribution function for flood damage corresponding to the flood control point k.

In application to a multi-reservoir system, the flood damage assessment index $CVaR_{k,\alpha}$ should be respectively established for sub-systems and the whole multi-reservoir system according to the downstream flood control point. Taking a two-reservoir system as an example, either in series or parallel as is shown in Fig. 2, if there exists two downstream flood control points in the two-reservoir system, the conditional value-at-risk index for the sub-system and the multi-reservoir system should be constructed based on Eq. (3). The sub-system consists of the 1st reservoir and its downstream flood control point i while the multi-reservoir system contains the 1st reservoir, 2nd reservoir, and the flood control point ii.

[Please insert Figure 2 here]

2.3 Optimization model for designing flood storage scheme

An optimization model with the purpose of maximizing the hydropower generation benefit is established to deduce the optimal FSCS for the whole multi-reservoir system.

2.3.1 Objective function

Maximization of hydropower benefits for the multi-reservoir system can be described by the annual average hydropower generation during the summer flood season as follows:

$$\max\ \overline{E}\left(x_1, x_2, \cdots, x_m\right) = \frac{1}{T}\sum_{j=1}^{T} E_j\left(x_1, x_2, \cdots, x_m\right) \tag{4}$$

where m is the number of reservoirs in the multi-reservoir system, x_i is the decision vector, representing the flood storage (m^3) of ith reservoir ($i = 1, 2, \cdots, m$), $E_j\left(\cdot\right)$ (kWh) represents the hydropower generation in the j^{th} year for the multi-reservoir system when the reservoir flood storage combination scheme is set as $x_1, x_2, \cdots, x_m$ ($j = 1, 2, \cdots, T$), $\bar{E}\left(\cdot\right)$ (kWh) represents the average annual hydropower generation for the multi-reservoir system during T years. In this study, the annual hydropower generation in the j^{th} year is calculated based on the observed data. The relationship between the average annual hydropower generation of the multi-reservoir system and the reservoir FSCS was established by conventional hydropower generation operation, which was evaluated in advance to reduce the computation time for searching for the optimal FSCS.

2.3.2 Constraints

(1) Conditional value-at-risk

$$CVaR_\alpha\left(x_1, x_2, \cdots, x_m\right) \le \beta_\alpha\left(x_1^*, x_2^*, \cdots, x_m^*\right) \tag{5}$$

where $\beta_\alpha\left(\cdot\right)$ (c m^3) is the conditional value-at-risk value with the current designed reservoir FSCS $x_1^*, x_2^*, \cdots, x_m^*$ corresponding to a confidence level α; $CVaR_\alpha\left(\cdot\right)$ (c m^3) is the conditional value-at-risk value with the optimal variables $x_1, x_2, \cdots, x_m$.

(2) Reservoir water balance equation:

$$V_{t+1} = V_t + \left(I_t - Q_t\right)\Delta t \tag{6}$$

where I_t (m^3/s) and Q_t (m^3/s) are the reservoir inflow and release during time period Δt, respectively, and V_t (m^3) is the reservoir storage at time t.

(3) Reservoir storage limits:

$$V_{\min} \le V_t \le V_{\max} \tag{7}$$

where $V_{\min}$ (m^3) and $V_{\max}$ (m^3) are the minimum and maximum allowable reservoir storages during flood season.

(4) Release capacity limits:

$$Q_t \le Q_{\max}\left(Z_t\right) \tag{8}$$

where $Q_{\max}\left(Z_t\right)$ (m^3/s) is the reservoir maximum discharge when the reservoir water level is Z_t (m) at time t.

(5) Initial condition:

$$V_{1,i} = V_{\max} - x_i \tag{9}$$

where $V_{1,i}$ is the initial reservoir storage value of the ith reservoir.

The trust region reflective algorithm in MATLAB's Optimization Toolbox, which returns the minimum of a nonlinear multivariate function, is chosen as the optimization algorithm for the proposed optimization model because of its high computing efficiency and easy use in the MATLAB programming environment.

3. Case study

3.1 Study area and data

The Hanjiang River has a basin area of about 159,000 km^2, which is the largest tributary of the Yangtze River in China. Fig. 3 shows that the Ankang reservoir and Danjiangkou reservoir are adjacent to each other in the main stream of the Hanjiang River, and the Ankang-Danjiangkou cascade reservoir system is selected as a case study. The summer flood season in the region is from late June to late August, and the characteristic parameter values of the multi-reservoir system are shown in Table 1.

[Please insert Figure 3 and Table 1 here]

The daily streamflow data from 1954-2010 were selected as the base period dataset for the Ankang Reservoir, Danjiangkou Reservoir, and Huangzhuang hydrological station. The design floods were derived using the magnification method with respect to several typical flood hydrographs, which is a common method used for the derivation of design floods in China (Huang et al., 2020; Liu et al., 2015b; Zhang et al., 2019). In this study, seven typical flood hydrographs (from 1957, 1978, 1980, 1981, 1983, 1989 and 2010) were selected to obtain the design floods.

3.2 $CVaR_{\alpha}$ under the present designed FSCS condition

For this two-reservoir system, the Ankang city is at the downstream flood control point of the upstream Ankang reservoir, while the Huangzhuang hydrological station is at the downstream flood control point of the whole Ankang-Danjiangkou reservoirs system (see Fig. 2). Therefore, the Ankang-Danjiangkou Reservoirs system is divided into a sub-system and a multi-reservoir system according to the two downstream flood control points. Specifically, the multi-reservoir system is composed of Ankang reservoir, Danjiangkou reservoir, and the Huangzhuang hydrological station, which is the focal point of this study.

The conditional value-at-risk $\beta_{HZ,\alpha}$ corresponding to the Ankang-Danjiangkou multi-reservoir's present designed FSCS during the summer flood season can be derived according to Eq. (5), that is $\beta_{HZ,0.99}$=6.479c billion yuan and $\beta_{HZ,0.999}$=6.481c billion yuan. The flood damage assessment index for the Ankang-Danjiangkou multi-reservoir system during the summer flood season corresponding to the different FSCS is named as $CVaR_{HZ,\alpha}$, and its upper limit value is set at the index $\beta_{HZ,\alpha}$ value of the present designed FSCS.

3.3 AMFS value for the reservoir in the two-reservoir system

The AMFS values of the Ankang and Danjiangkou reservoirs are respectively deduced through the following schemes: (i) The flood storage value of the Danjiangkou reservoir is fixed at its present designed value while the Ankang reservoir's flood storage changes. (ii) The flood storage value of the Ankang reservoir is fixed at its present designed value while the Danjiangkou reservoir's flood storage changes. With the index $\beta_{HZ,\alpha}$ corresponding to the present designed FSCS as the upper limit value for the flood damage assessment index $CVaR_{HZ,\alpha}$, the AMFS values of the Ankang and Danjiangkou reservoirs in the two-reservoir system can be derived to be equal to 0.36 billion m^3 and 10.59 billion m^3, respectively. Moreover, the allowable minimum total flood storage value of this two-reservoir system is 10.95 billion m^3.

4. Results and discussions

4.1 Feasible area for the FSCSs in the two-reservoir system

4.1.1 Boundary of the feasible FSCSs when fixing the two-reservoir system's total flood storage (TFS)

The relationship between the flood damage assessment index and flood storage values of the two reservoirs is shown in Fig. 4, $CVaR_{\alpha}$ values for a given confidence level α =0.99 first decrease significantly as the flood storage of the Danjiangkou reservoir (or Ankang reservoir) increases and then stabilize at a certain value when the flood storage value of the Ankang reservoir (or Danjiangkou reservoir) is fixed. However, the loss function of the flood damage for the two-reservoir system in this study is the expression of three variables, i.e., the Ankang reservoir's flood storage, the Danjiangkou reservoir's flood storage, and the inflow magnitude, thus, it is difficult to directly judge the relationship between the loss function L_{HZ} (or the $CVaR_{HZ,\alpha}$ value) and the FSCS of the two reservoirs in the system because of the high dimension.

[Please insert Figure 4 here]

In order to clarify the influence of different FSCSs on the flood damage assessment index $CVaR_{\alpha}$, Scenario 1 is established as follows: The Ankang-Danjiangkou Reservoirs system's TFS during the summer flood season is fixed at a constant value, and then the FSCSs of the two reservoirs change. Considering that present designed flood storage values of the Ankang and Danjiangkou reservoirs are 0.36 billion m^3 and

11.00 billion m^3 in the summer flood season, respectively, TFS value of this two-reservoir system is assumed to be fixed at 11.36 billion m^3. Then, the flood storage value of the Ankang reservoir changes from 0.01 billion m^3 to 1.41 billion m^3, and the Danjiangkou reservoir's flood storage is adjusted with the change of the Ankang reservoir's flood storage.

[Please insert Figure 5 here]

Fig. 5(a) and Table 2 show the results of Scenario 1 when FSCSs for the two reservoirs change, which are numbered from 1 to 29 in sequence. The flood damage assessment index $CVaR_{HZ,\alpha}$ of this multi-reservoir system during the summer flood season presents a decreasing trend in schemes 1 to 8, keeps at a constant value in schemes 8 to 16, and finally increases in schemes 16 to 29. Therefore, one point can be drawn from the analysis results in Scenario 1, that is, the $CVaR_{HZ,\alpha}$ value of the multi-reservoir system varies if FSCSs of the Ankang and Danjiangkou reservoirs change even though the TFS value is fixed at a constant value. In addition, FSCSs from schemes 8 to 16 are the feasible schemes in the summer flood season, which are called feasible intervals of FSCSs, with conditional value-at-risk $\beta_{HZ,\alpha}$ values under the present designed FSCS (see Section 3.2) as the upper limit values.

In order to explore the boundary condition of FSCS's feasible intervals for multi-reservoir systems, several FSCSs are added. The left boundary of the feasible interval for FSCSs is obtained where the Ankang reservoir's flood storage value is equal to 0.36

billion m^3 (Fig. 5(b)). In Fig. 5(c), the right boundary of the feasible interval for FSCSs is acquired where the Danjiangkou reservoir's flood storage value is 10.59 billion m^3. It should be emphasized that the terms "left" and "right" here correspond to the situation for convenience of expression, of which the flood storage value of Ankang reservoir during the summer flood season changes from 0.01 billion m^3 to 1.41 billion m^3 and the flood storage value of Danjiangkou reservoir changes accordingly to keep TFS at a constant value.

If TFS of this two-reservoir system ranges from 10.95 billion m^3 (the allowable minimum TFS derived in Section 3.3) to 11.36 billion m^3 (the present designed TFS given in Table 1), the feasible FSCSs can be represented as a triangle area (Fig. 6), and the red line corresponds to the feasible interval of FSCSs in Scenario 1 (i.e., scheme 8 to 16 in Fig. 5). It should be noted that V_{AK} and V_{DJK} respectively represent Ankang and Danjiangkou reservoirs' flood storage values in Figure 6.

[Please insert Figure 6 here]

4.1.2 Discussion on the shape of feasible FSCSs area

On the basis of Scenario 1, the flood damage assessment index $CVaR_{\alpha}$ of the two-reservoir system corresponding to that the TFS in the summer flood season successively fixed in the range of 10.95 billion m^3 to 15.13 billion m^3 is estimated, the result of which is shown in Figure 7(a). If the TFS of multi-reservoir system is smaller than and equal to 12.024 billion m^3, the shape for feasible FSCSs area is a triangle; if the TFS of this two-reservoir system is larger than 12.024 billion m^3, the shape of feasible FSCSs area turns into a right-angled trapezoid because of the Ankang reservoir's flood storage value reaches its maximum. In addition, V_{AK}^{D} and V_{DJK}^{D} are the Ankang and Danjiangkou reservoirs' AMFS values, respectively, while V_{AK}^{U} and V_{DJK}^{U} are the Ankang and Danjiangkou reservoirs' maximum flood storage values which are determined by the characteristic of the reservoir itself.

Only two scenario results (shown in Fig. 7(b) and Fig. 7(c)) are selected for further analysis because of the obvious regularity of the all scenarios and the inconvenience of displaying them all. Fig. 7(b) shows $CVaR_{HZ,\alpha}$ values when the multi-reservoir system's TFS is fixed at 11.67 billion m^3 (i.e., the gray line in Fig. 7(a)), and furthermore the left and right boundaries of the feasible interval of FSCSs are obtained at AMFS values of the Ankang and Danjiangkou reservoirs, respectively. The value of $CVaR_{HZ,\alpha}$ first decreases as the Ankang reservoir's flood storage increases, and then goes through an interval to stabilize, which is called the feasible interval of the FSCSs. However, the $CVaR_{HZ,\alpha}$ value increases significantly when the flood storage of Ankang reservoir is larger (that is, the corresponding Danjiangkou reservoir's flood storage is smaller). The above results show the same regular pattern as Fig. 5(a). Fig. 7(c) presents $CVaR_{HZ,\alpha}$ values when the TFS is fixed at 12.17 billion m^3 (i.e., the green line in Fig. 7(a)), and the left boundary of the feasible interval of FSCSs is the Ankang reservoir's AMFS. However, the right boundary of feasible FSCSs interval does not exist because the flood storage value of Danjiangkou reservoir is always larger than its AMFS value. It should be noted that the result given in Fig. 7(c) is a special case because of the big difference in the flood capacity levels of two reservoirs, which is not inconsistent with the conclusion in Section 4.1.1.

[Please insert Figure 7 here]

Comparison results between the shape of feasible FSCSs area derived by the $CVaR_{\alpha}$ index (see the area filled with green stippled lines in Fig. 8(d)) and the traditional risk-based method (see the area filled with the blue triangle in Fig. 8(d)) are shown in Fig. 8, and the detailed certification process is given in Appendix A. Feasible FLWLs of this study is inferred from the design level while that of Chen et al. (2012) is inferred by considering the streamflow forecast information at the real-time operating level. Furthermore, previous studies, including Chen et al. (2012), about the feasible FLWLs paid more attention to the upper limit of the reservoir water level during the flood season by making full use of the forecast information, and the static FLWL was conventionally used as the lower limit in order not to sacrifice the power generation benefit. However, the lower limit value of feasible FLWL combination deduced by the flood damage assessment index $CVaR_{\alpha}$ is not as small as possible because of the flood capacity restraints for the whole multi-reservoir system and each sub-system.

[Please insert Figure 8 here]

4.1.3 Discussion on the sensitivity of $CVaR_\alpha$ to the flood storage of each reservoir

Scenario 2 aims to analyze the influence of each reservoir's flood storage value on the $CVaR_\alpha$ value of the multi-reservoir system as follows: The Ankang (Danjiangkou) reservoir's flood storage value is fixed as a constant value, while the flood storage value of Danjiangkou (Ankang) reservoir changes, and vice versa. Figure 9 shows the results of Scenario 2, and the change in $CVaR_{HZ,\alpha}$ value caused by the unit flood storage change of Danjiangkou reservoir is greater when compared with that of the Ankang reservoir. Therefore, the $CVaR_{HZ,\alpha}$ value is more sensitive to changes in the flood storage of Danjiangkou reservoir than that of Ankang reservoir because of the difference of reservoir flood storage capacity and upstream and downstream positional relationship of the two reservoirs. Furthermore, the discussion on a five-reservoir system is given in Appendix B, and the conclusion can be drawn as following: a) if the positional relationship between the two reservoirs is parallel, the greater the reservoir flood control capacity, the lower the sensitivity of $CVaR_\alpha$ to the amplitude of the flood storage value; b) if the two reservoirs are in series position, the sensitivity of $CVaR_\alpha$ to the amplitude of flood storage value for the downstream reservoir is higher than that for the upstream reservoir.

[Please insert Figure 9 here]

4.2 Tarde-offs between flood damage and hydropower benefit by considering the $CVaR_{\alpha}$ index

The flood control benefits provided by the feasible FSCSs of Ankang-Danjiangkou Reservoirs system are the same from the perspective of the flood damage assessment $CVaR_{\alpha}$ index. However, the trade-offs between activities of flood control and hydropower generation need to be further explored (Rheinheimer et al., 2016; Wang et al., 2021), and then an optimization model (Section 2.3) was used to search for the optimal FSCS from the feasible FSCSs.

[Please insert Figure 10 here]

The TFS of the two-reservoir system is fixed at its present designed value (i.e., 11.36 billion m^3) in Figure 10, the red points represent the hydropower generation benefit (i.e., plus character) and flood damage assessment index $CVaR_{HZ,\alpha}$ values (i.e., triangular and square characters) corresponding to the optimal FSCS. Further discussions are conducted from the following aspects:

(1) The relationship between the hydropower generation benefit and the flood damage assessment index $CVaR_{\alpha}$ is not monotonic. Thus, the feasible FSCSs of the multi-reservoir system taking the constraint of $CVaR_{\alpha}$ value into consideration should be used as a prerequisite for seeking the optimal solution where the maximum hydropower generation benefit is obtained.

(2) Fig. 10 reveals the larger the flood storage value of Ankang reservoir, the lesser

the hydropower generation benefit of two-reservoir system. In other words, if the TFS of the two-reservoir system is fixed, the flood storage is suggested allocated to the Danjiangkou reservoir (the downstream reservoir in this two-reservoir system) with the purpose of maximizing the hydropower generation benefit.

[Please insert Figure 11 here]

Optimal results when the Ankang-Danjiangkou Reservoirs system's TFS in the summer flood season is successively fixed in the range of 10.95 billion m^3 to 13.95 billion m^3 are shown in Figure 11. It should be noted that all the FSCSs shown in Fig. 11 satisfy the constraint of the flood assessment $CVaR_\alpha$ index. Several points can be drawn as follows:

(1) The maximization value of the hydropower generation benefit was achieved where both the Ankang and Danjiangkou reservoirs' flood storage values are equal to their AMFS values (the red asterisk point shown in Fig. 11): The Ankang reservoir's flood storage value is 0.36 billion m^3 while the Danjiangkou reservoir's flood storage value is 10.59 billion m^3.

(2) The optimal results in Fig. 11 reveal the conflict relationship between the hydropower generation benefit and the TFS, i.e., the larger the TFS, the lesser the hydropower generation benefit of the two-reservoir system. Moreover, there is also a monotonous decreasing relationship between the hydropower generation benefit and each single reservoir's flood storage value in the Ankang-Danjiangkou Reservoirs

system.

5. Conclusions

This paper proposed a new joint optimization method for design and operation of multi-reservoir system considering the index $CVaR_\alpha$, and focused on analyzing the mutual feedback mechanism between the flood storage of each reservoir in the systems. The main findings are made as follows:

(1) The $CVaR_\alpha$ values of the multi-reservoir system vary if FSCSs of the Ankang and Danjiangkou reservoirs are different even though the TFS is set to the same constant value. The feasible area of FSCSs derived by considering the $CVaR_\alpha$ index can be described as a triangle, and the boundary of this feasible interval is determined by the AMFS value of reservoirs in the mixed multi-reservoir system.

(2) The contribution degree of each reservoir to flood control benefits can be summarized as following: a) if the reservoirs are in parallel positions, the greater the reservoir flood control capacity, the lower the sensitivity of $CVaR_\alpha$ to the amplitude of the flood storage value; b) if the reservoirs are in series position, the sensitivity of $CVaR_\alpha$ to the amplitude of flood storage value for the downstream reservoir is higher than that for the upstream reservoir.

(3) The relationship between the hydropower generation benefit and the flood damage assessment index $CVaR_\alpha$ is not monotonic. The joint optimal design of multi-reservoir system FSCS can be deduced by establishing an optimal model with the purpose of maximizing the hydropower generation benefit for the whole multi-reservoir

system. The global optimal solution of the hydropower generation benefit was achieved when all reservoirs' flood storage values were respectively equal to their AMFS.

It's worth noting that the index $CVaR_\alpha$ can be applied in multi-reservoir system (more than two reservoirs) with following steps: (i) System division is conducted according to the protection object of downstream flood control point (refer to Section 2.2). (ii) Flood damage assessment indexes $CVaR_\alpha$ for the sub-system and the whole multi-reservoir system are respectively established to deduce the AMFS of each reservoir and the FSCSs for the multi-reservoir system (refer to Section 4.1.1). In addition, the relationship between any two reservoirs in the system can be derived by dividing the multi-reservoir system into several two-reservoir sub-systems. The establishment of the flood damage assessment index by taking the concept of $CVaR_\alpha$ into consideration can be more accurate if the relevant actual socioeconomic data is available. For example, the unit cost c of loss function is assumed constant, which can be determined or further researched when the economic assessment of flood control losses for the specific study area is provided. The $CVaR_\alpha$ value would be applied in the multi-objective optimization operation fields by coupling the benefit targets (such as hydropower generation benefit, water supply benefit) since it can reflect the possible flood damage loss. Moreover, the feasible interval of the flood storage for a multi-reservoir system derived in this paper can help understand the research on the dynamic control of the FLWLs in multi-reservoir systems.

Author statement

Zhang Xiaoqi: Conceptualization, Methodology, Software Writing- Original Draft **Liu Pan:** Conceptualization, Supervision, Writing- Review & Editing **Feng Maoyuan:** Methodology, Validation, Writing- Review & Editing **Xu Chong-Yu:** Supervision, Writing- Review & Editing **Cheng Lei:** Validation, Writing- Review & Editing **Gong Yu:** Data Curation, Writing- Review & Editing

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A: Derivation for the shape of the feasible FSCSs area by the $CVaR_\alpha$ index and the traditional risk-based method

The feasible FSCSs of the two-reservoir system obtained through the flood damage assessment index $CVaR_\alpha$ are shown as a triangle area (see Figure 7(a)), which is simplified into a schematic diagram (see Figure 8(a)) for the convenience of explanation. The feasible FSCSs area is determined by the following variables: (i) the allowable maximum flood storage of reservoir A, V_A^U; (ii) the allowable minimum flood storage (AMFS) of reservoir A, V_A^D, which corresponds to the static control of FLWL for reservoir A; (iii) the allowable maximum flood storage of reservoir B, V_B^U; (iv) the AMFS of reservoir B, V_B^D, which corresponds to the static control of FLWL for reservoir B. In addition, the terms V_A^U and V_B^U are both assumed to not exceed their maximum storage capacity constraints. The coordinates of the points *f* and *g* of the red line in Fig. 8(a) are respectively $\left(V_B^D, V_A^U\right)$ and $\left(V_B^U, V_A^D\right)$, which satisfy the curve equation as follows:

$$V_A + V_B \cdot K = M \tag{A1}$$

where V_A and V_B are the reservoir flood storage values of reservoir A and B, respectively, and the term $\left(V_B, V_A\right)$ is the point on the curve Equation A1. Parameter K is the slope of the linear function, and parameter M is the intercept from the perspective of mathematical theory. Then, parameters $K = \left(V_A^U - V_A^D\right) / \left(V_B^U - V_B^D\right) > 0$ and $M = \left(V_A^U \cdot V_A^D - V_B^U \cdot V_B^D\right) / \left(V_B^U - V_B^D\right)$ can be deduced by solving the equation A2.

$$\begin{cases} V_A^U + V_B^D \cdot K = M \\ V_A^D + V_B^U \cdot K = M \end{cases} \tag{A2}$$

In Figure 8(b), the feasible FLWL combination schemes of the two-reservoir system derived by Chen et al. (2012) is described as a Fan-shaped area, which is deduced through the traditional risk-based approach. The feasible FLWLs area is determined by the following variables: (i) the allowable maximum FLWL of reservoir A, Z_A^*, which is determined by the dynamic control of FLWL; (ii) the allowable minimum FLWL of reservoir A, Z_A^U, which is equal to the static control of FLWL; (iii) the allowable maximum FLWL of reservoir B, Z_B^*, which is determined by the dynamic control of FLWL; (iv) the allowable minimum FLWL of reservoir B, Z_B^U. In order to better compare the area size acquired by these two different methods, the feasible FSCSs of the two-reservoir system in Figure 8(a) should be converted to a feasible FLWLs which is unified with the characterization variable with Figure 8(b). It should be noted that the relationship between the reservoir water level Z_j and the flood storage V_j is assumed as a power function form (Mohammadzadeh-Habili and Heidarpour, 2010; Zhang et al., 2019b) and is shown in Equation A3.

$$Z_j = a_j S_j^{b_j} = a_j \left(V_{j,n} - V_j \right)^{b_j} \quad \text{(A3)}$$

where S_j is the water stored in the reservoir, $V_{j,n}$ is the reservoir storage corresponding to its normal pool level, a_j and b_j are parameters of the power function, $a_j > 1$ and $0 < b_j < 1$, $j = A \; or \; B$.

The variables Z_A^D and Z_A^U are respectively the reservoir water levels corresponding to the reservoir flood storage values V_A^U and V_A^D In Fig. 8(c), and the key point is to clarify the position of the envelope of the feasible FLWLs area. Based on Equations A1 and A3, the relationship between variables and can described as Eq. (A4).

$$Z_A = a_A \cdot \left[H - c_B \cdot \left(Z_A \right)^{d_B} \right]^{b_A} > 0 \quad \text{(A4)}$$

where $a_A > 1$, $0 < b_A < 1$, $c_B = K \cdot \left(\frac{1}{a_B} \right)^{\frac{1}{b_B}} > 0$, $d_B = \frac{1}{b_B} > 1$, and $H = V_{A,n} + V_{B,n} - M > 0$.

The derivation results for signs of $\partial Z_A \big/ \partial Z_B$ and $\partial^2 Z_A \big/ \partial \left(Z_B\right)^2$ are shown in Eqs. (A5)-(A6), indicating that ∂Z_A decreases when ∂Z_B increases and that ∂Z_B has a diminishing marginal contribution to ∂Z_A. Therefore, the type of envelope in Fig. 8(c) should be the type I (green dotted line).

$$\frac{\partial Z_A}{\partial Z_B} = -a_A b_A c_B d_B \left(Z_B\right)^{d_B-1} \cdot \left[H - c_B \cdot \left(Z_B\right)^{d_B}\right]^{b_A-1} < 0 \tag{A5}$$

$$\begin{aligned} \frac{\partial^2 Z_A}{\partial Z_B^2} &= -a_A b_A c_B d_B \left(d_B - 1\right)\left(Z_B\right)^{d_B-2} \cdot \left[H - c_B \cdot \left(Z_B\right)^{d_B}\right]^{b_A-1} \\ &\quad + a_A b_A c_B d_B \left(Z_B\right)^{d_B-1} \cdot \left(b_A - 1\right) \cdot \left[H - c_B \cdot \left(Z_B\right)^{d_B}\right]^{b_A-2} \cdot c_B \cdot d_B \left(Z_B\right)^{d_B-1} \\ &= -\frac{a_A b_A c_B d_B \left(Z_B\right)^{d_B-2}}{\left[H - c_B \cdot \left(Z_B\right)^{d_B}\right]^{1-b_A}} \cdot \left\{ \left(d_B - 1\right) + \frac{\left(1-b_A\right) c_B d_B \left(Z_B\right)^{d_B}}{H - c_B \cdot \left(Z_B\right)^{d_B}} \right\} < 0 \end{aligned} \tag{A6}$$

The comparison results between the feasible FLWLs obtained by the index $CVaR_\alpha$ and the traditional flood risk probability, respectively are shown in Fig. 8(d).

Appendix B: Discussion on the sensitive of $CVaR_\alpha$ to the flood storage of each reservoir in a five-reservoir system

The flood damage assessment index $CVaR_\alpha$ for the multi-reservoir system proposed in Section 2.2 is applied in a five-reservoir system in order to identify the sensitive factor for the relationship between the $CVaR_\alpha$ value and the reservoir flood storage. A five-reservoir system, which includes the Ankang, Pankou, Danjiangkou, Sanliping and Yahekou reservoirs, is extended from the two-reservoir system in Section 4.1.3, and the location relationship among these five reservoirs is shown in Fig. B1.

The results of relationship between the flood damage assessment index $CVaR_\alpha$

value and Pankou/Sanliping/Yahekou reservoirs's flood storage value are shown in Fig. B2 (Results for Ankang/Danjiangkou reservoirs are shown in Fig. 9). The flood control capacity of the five reservoirs in descending order are: Danjiangkou, Pankou, Ankang, Yahekou, Sanliping reservoirs. As shown in Fig. B2, the sensitivity of $CVaR_{\alpha}$ to the amplitude of the flood storage value for the five reservoirs in descending order are: Sanliping, Yahekou, Danjiangkou, Ankang, Pankou reservoirs. Several points can be concluded as follows:

a) When the reservoirs are in parallel relationship, the greater the reservoir flood control capacity, the lower the sensitivity of $CVaR_{\alpha}$ to the amplitude of the flood storage value. Facing the common downstream flood control point, parallel reservoirs affect the flood control effect by separately undertaking the flood regulation of the river channels in their respective locations. Therefore, the reservoir with the smaller flood control capacity should be paid more attention in the case of encountering the same design frequency of inflow. Furthermore, it is recommended to allocate the new unit storage to the reservoir with smaller flood control capacity among the parallel reservoirs when the system's total flood storage (TFS) needs to increase, so as to greatly reduce the potential flood damage (i.e., the $CVaR_{\alpha}$ value). Conversely, if the TFS decreases, it is recommended to adjust the reservoir with larger flood control capacity among the parallel reservoirs, which can minimize the increase in the multi-system's potential flood control losses.

b) If the reservoirs are in series position, the sensitivity of $CVaR_\alpha$ to the amplitude of flood storage value for the downstream reservoir is higher than that for the upstream reservoir. For series reservoirs, the downstream reservoir is to regulate both the release from the upstream reservoir and the interval inflow. Therefore, compared with the upstream reservoir, the downstream reservoir has a greater sphere of influence in regulation of flood control points in the multi-reservoir system. If the TFS increases, the added unit storage is advised to the downstream reservoir, which can greatly lessen the potential flood damage. On the contrary, if the TFS decreases, it is recommended to reduce the upstream reservoir's flood storage with the purpose of decreasing the increment of the $CVaR_\alpha$ value.

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Tables

Table 1. Characteristic parameter values for the Ankang-Danjiangkou reservoirs.

Reservoir	Parameter	Unit	Value
	Flood storage in the summer flood season	billion m^3	0.36
Ankang	FLWL during the summer flood season	m	325.0
reservoir	Normal pool level	m	330.0
	Dead water level	m	305.0
	Flood storage in the summer flood season	billion m^3	11.0
Danjiangkou	FLWL during the summer flood season	m	160.0
reservoir	Normal pool level	m	170.0
	Dead water level	m	155.0

Table 2. Calculation results of Scenario 1: the total flood storage of the Ankang-Danjiangkou Reservoirs system during the summer flood season is fixed at a constant value, and then the FSCSs of the two reservoirs changes.

Scheme number	Ankang reservoir's flood storage (10^8 m^3)	Danjiangkou reservoir's flood storage (10^8 m^3)	Total flood storage (10^8 m^3)	$CVaR_{HZ,\alpha}$(c 10^8 m^3)		Scheme number	Ankang reservoir's flood storage (10^8 m^3)	Danjiangkou reservoir's flood storage (10^8 m^3)	Total flood storage (10^8 m^3)	$CVaR_{HZ,\alpha}$(c 10^8 m^3)	
				α=0.999	α=0.99					α=0.999	α=0.99
1	0.10	113.50	113.6	65.44	65.19	**16**	**7.60**	**106.00**	113.6	**64.81**	**64.79**
2	0.60	113.00	113.6	65.37	65.13	17	8.10	105.50	113.6	64.87	64.80
3	1.10	112.50	113.6	65.30	65.08	18	8.60	105.00	113.6	64.88	64.82
4	1.60	112.00	113.6	65.23	65.01	19	9.10	104.50	113.6	64.91	64.83
5	2.10	111.50	113.6	65.16	64.95	20	9.60	104.00	113.6	64.92	64.85
6	2.60	111.00	113.6	65.09	64.88	21	10.10	103.50	113.6	64.92	64.86
7	3.10	110.50	113.6	65.01	64.82	22	10.60	103.00	113.6	64.94	64.88
8	**3.60**	**110.00**	**113.6**	**64.81**	**64.79**	23	11.10	102.50	113.6	64.96	64.90
9	**4.10**	**109.50**	**113.6**	**64.81**	**64.79**	24	11.60	102.00	113.6	64.96	64.91
10	**4.60**	**109.00**	**113.6**	**64.81**	**64.79**	25	12.10	101.50	113.6	65.10	64.96
11	**5.10**	**108.50**	**113.6**	**64.81**	**64.79**	26	12.60	101.00	113.6	65.17	65.02
12	**5.60**	**108.00**	**113.6**	**64.81**	**64.79**	27	13.10	100.50	113.6	65.24	65.08
13	**6.10**	**107.50**	**113.6**	**64.81**	**64.79**	28	13.60	100.00	113.6	65.31	65.14
14	**6.60**	**107.00**	**113.6**	**64.81**	**64.79**	29	14.10	99.50	113.6	65.38	65.20
15	**7.10**	**106.50**	**113.6**	**64.81**	**64.79**						

Figure captions

Fig. 1. Flowchart of proposed method.

Fig. 2. Sketch of the two types for the two-reservoir system.

Fig. 3. Location of the Hanjiang basin and gauge stations.

Fig. 4. Relationship between flood storage of the two reservoirs and $CVaR_{\alpha}$ values: (a) Three-dimensional perspective; (b) Projection to the V_{DJK}-$CVaR_{\alpha}$; (c) Projection to the V_{AK}-$CVaR_{\alpha}$.

Fig. 5. Results of Scenario 1: the total flood storage of the Ankang-Danjiangkou multi-reservoir system during the summer flood season is fixed at a constant value (i.e., 11.36 billion m^3), and then the FSCSs of the two reservoirs changes.

Fig. 6. Results of the feasible area for the FSCSs in the multi-reservoir system when the total flood storage value ranges from 10.95 to 11.36 billion m^3.

Fig. 7. Results of the $CVaR_{HZ,\alpha}$ values for the multi-reservoir system during the summer flood season: (a) when the total flood storage value V changes from 10.95 to 15.13 billion m^3; (b) when the total flood storage value is fixed at 11.67 billion m^3; (c) when the total flood storage value is fixed at 12.17 billion m^3.

Fig. 8. Schematic diagram of feasible FLWLs/FSCSs area.

Fig. 9. Results of Scenario 2: the flood storage of the Ankang/Danjiangkou reservoir changes while another reservoir's flood storage is fixed at its the present designed value.

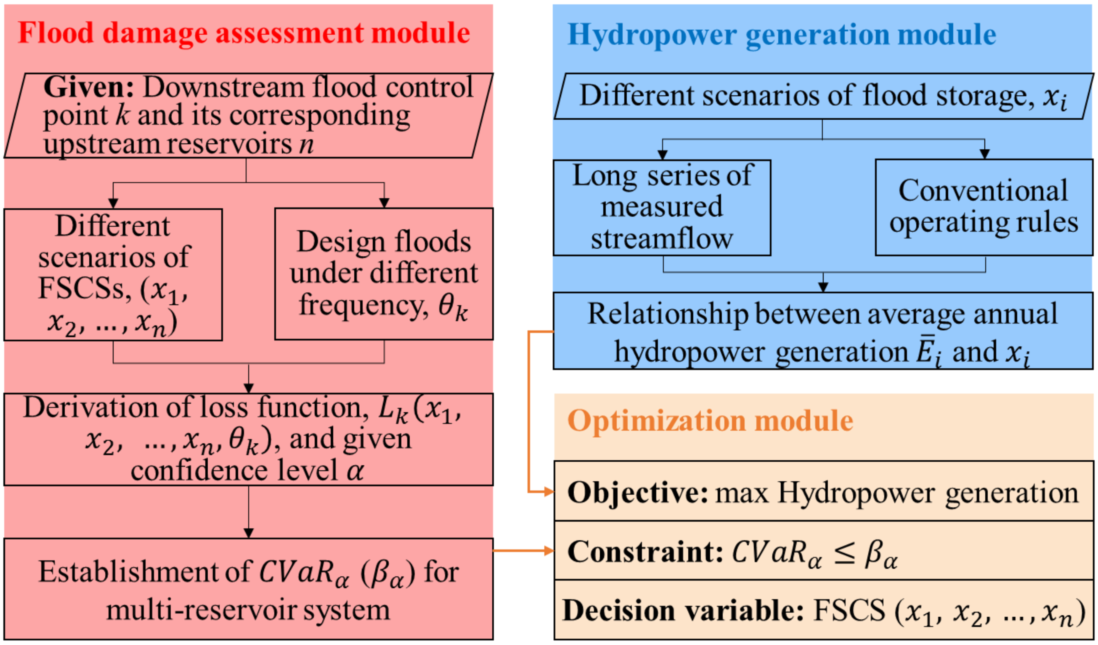
Fig. 10. Optimal result when the total flood storage value is fixed at 11.36 billion m^3.

Fig. 11. Optimal results when the total flood storage value V is successively fixed in the range of 10.95 billion m^3 to 13.95 billion m^3: (1) The solid lines in the horizontal plane represent the different FSCSs, and the circle on each solid line is the optimal FSCS when the term V is fixed at a constant value; (2) The asterisk points are the vertical projection for the curved surface when the Ankang Reservoir's flood storage is fixed at 0.36 billion m^3.

Fig. B1. Schematic diagram of the location for the five reservoirs.

Fig. B2. Results of relationship between the flood damage assessment index $CVaR_{\alpha}$ value and Pankou/Sanliping/Yahekou reservoir's flood storage value.

737 Figures

738 

739 **Fig. 1.** Flowchart of proposed method.

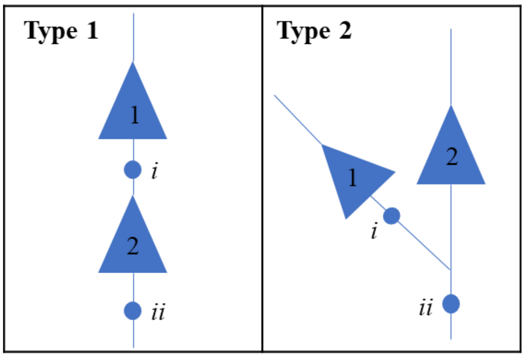


Fig. 2. Sketch of the two types for the two-reservoir system.

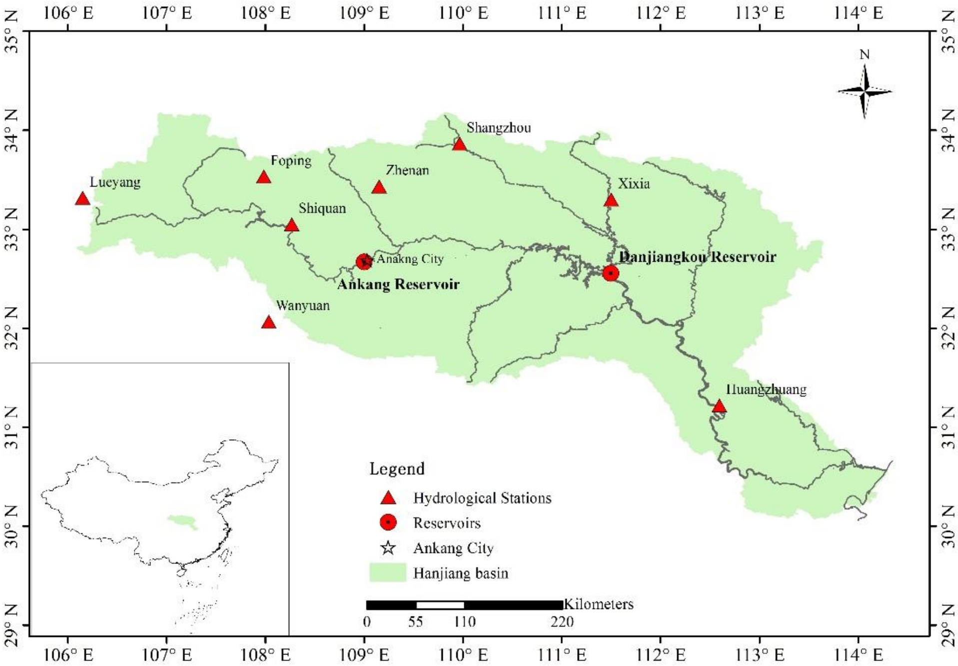


Fig. 3. Location of the Hanjiang basin and gauge stations.

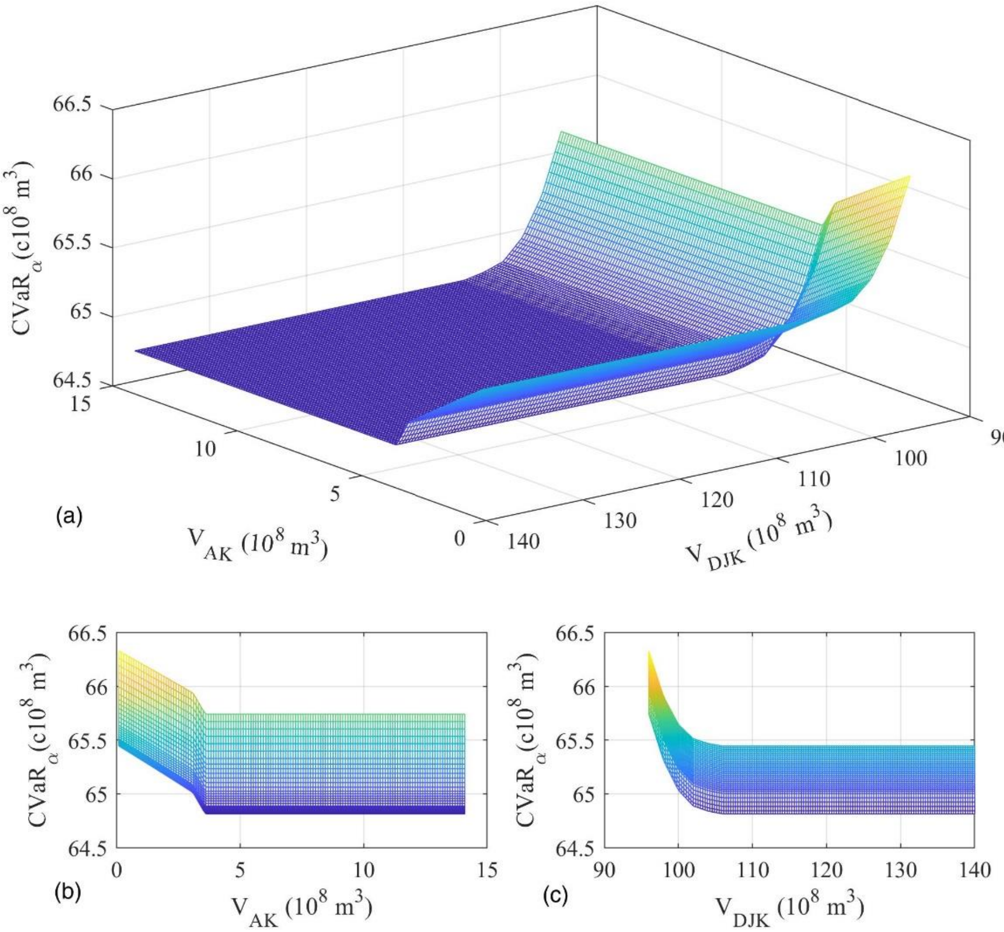


Fig. 4. Relationship between flood storage of the two reservoirs and $CVaR_{\alpha}$ values: (a) Three-dimensional perspective; (b) Projection to the V_{DJK}-$CVaR_{\alpha}$; (c) Projection to the V_{AK}-$CVaR_{\alpha}$.

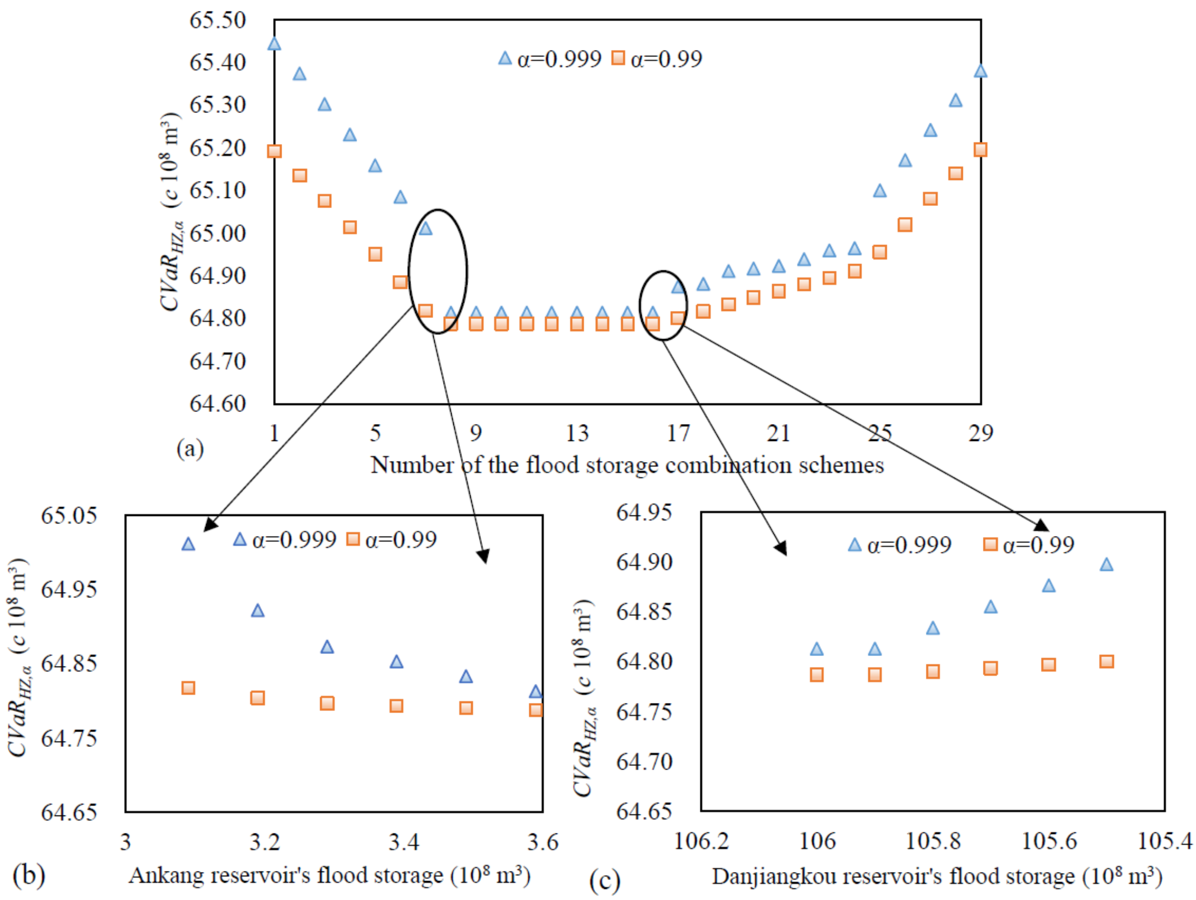


Fig. 5. Results of Scenario 1: the total flood storage of the Ankang-Danjiangkou multi-reservoir system during the summer flood season is fixed at a constant value (i.e., 11.36 billion m^3), and then the FSCSs of the two reservoirs changes.

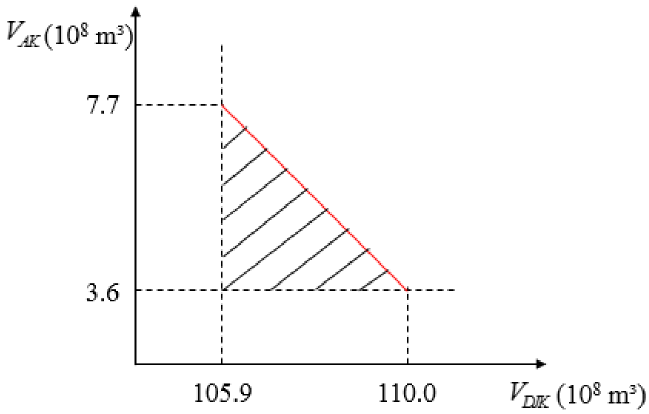


Fig. 6. Results of the feasible area for the FSCSs in the multi-reservoir system when the total flood storage value ranges from 10.95 to 11.36 billion m^3.

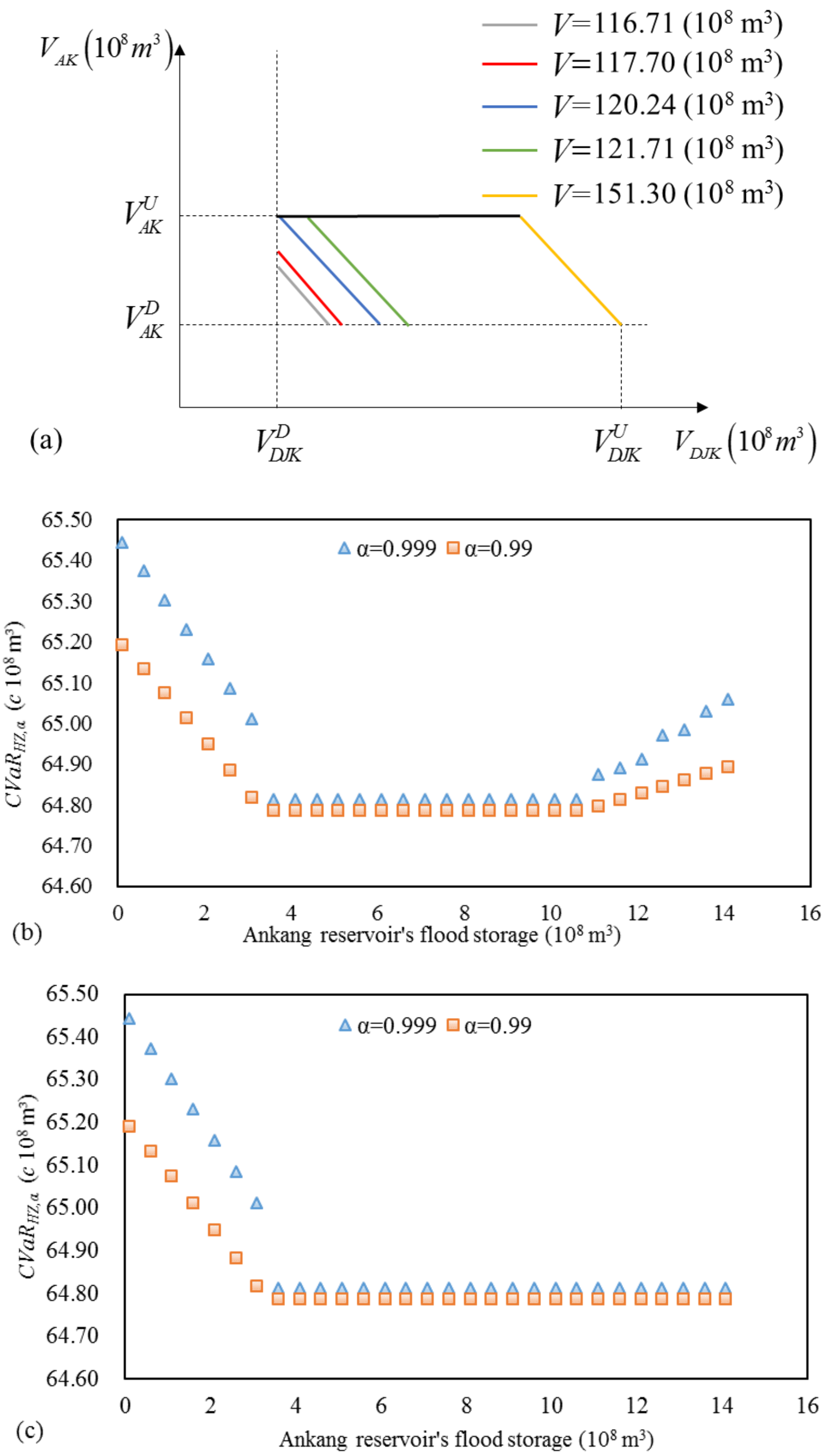


Fig. 7. Results of the $CVaR_{HZ,\alpha}$ values for the multi-reservoir system during the summer flood season: (a) when the total flood storage value V changes from 10.95 to 15.13 billion m^3; (b) when the total flood storage value is fixed at 11.67 billion m^3; (c) when the total flood storage value is fixed at 12.17 billion m^3.

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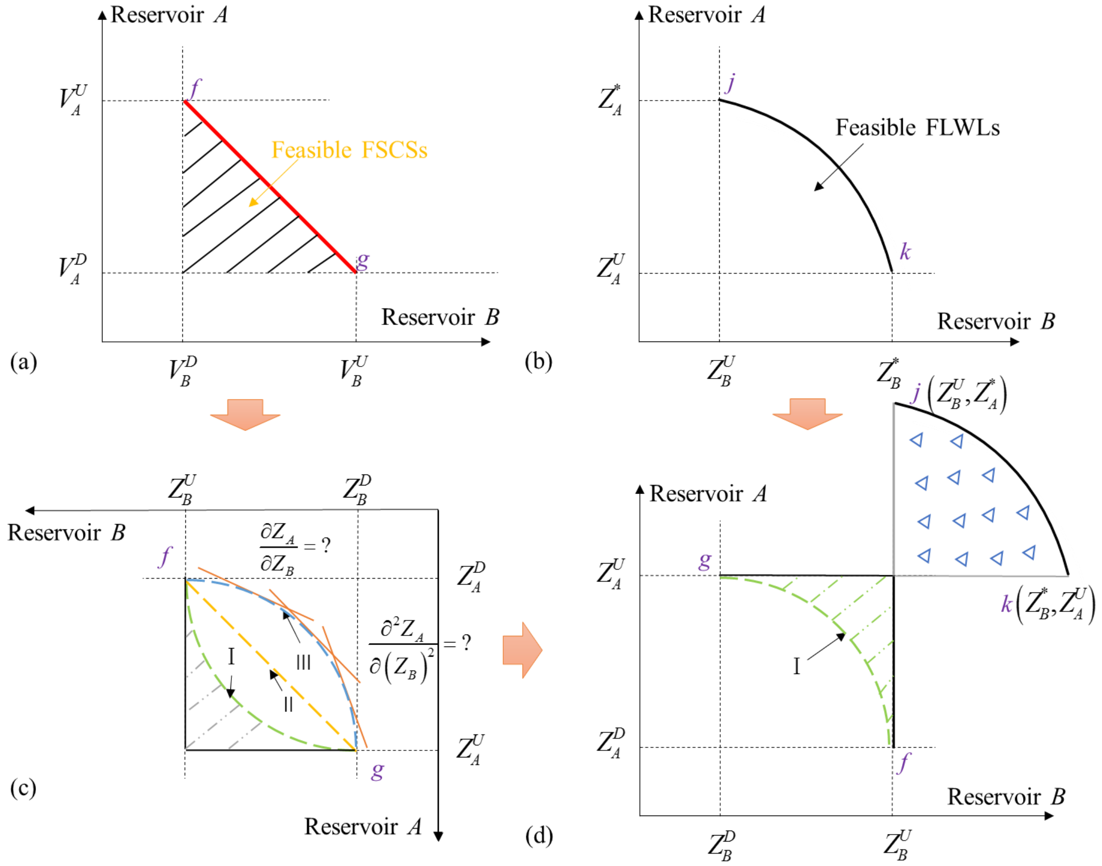
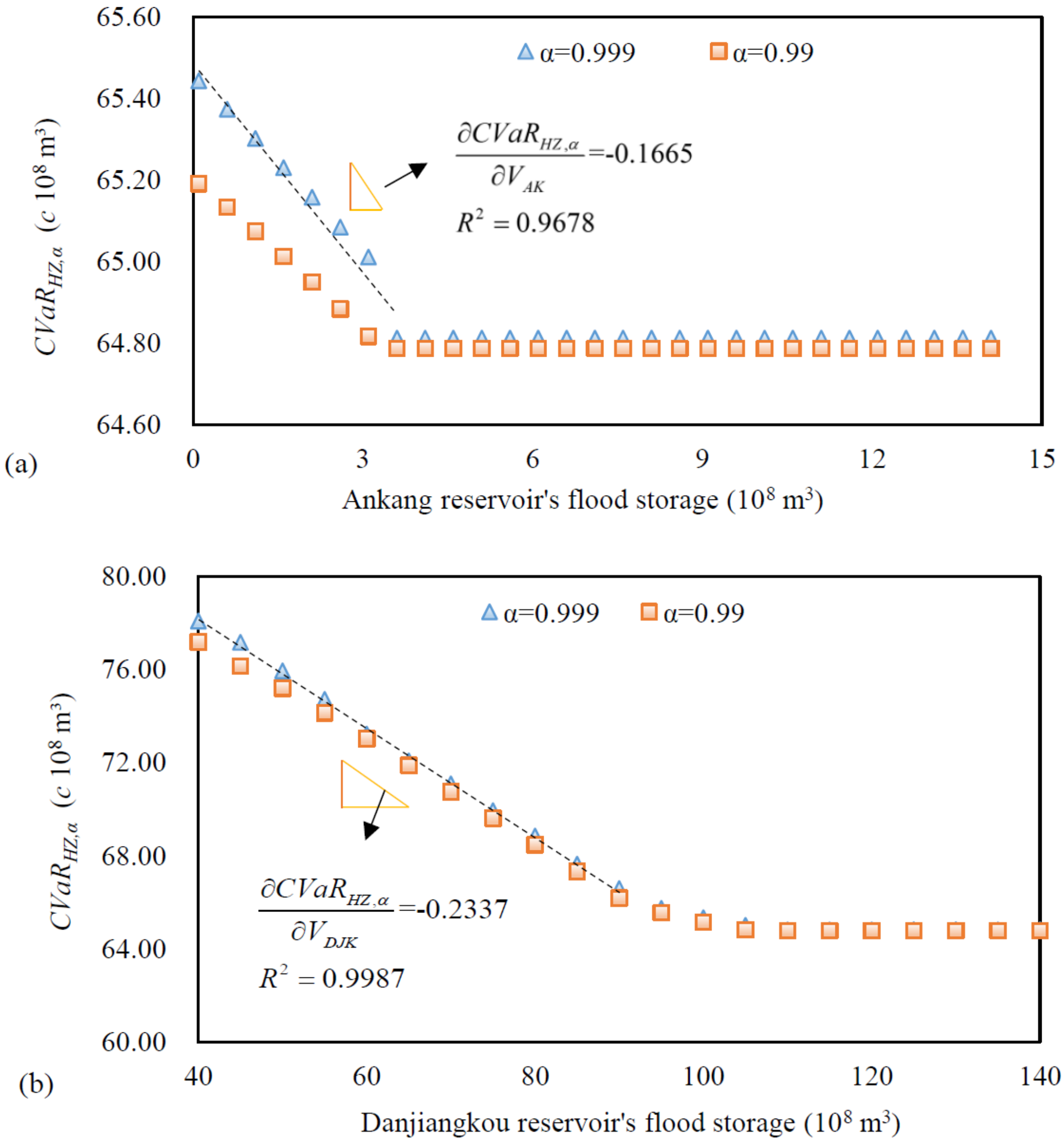


763 **Fig. 8.** Schematic diagram of feasible FLWLs/FSCSs area.



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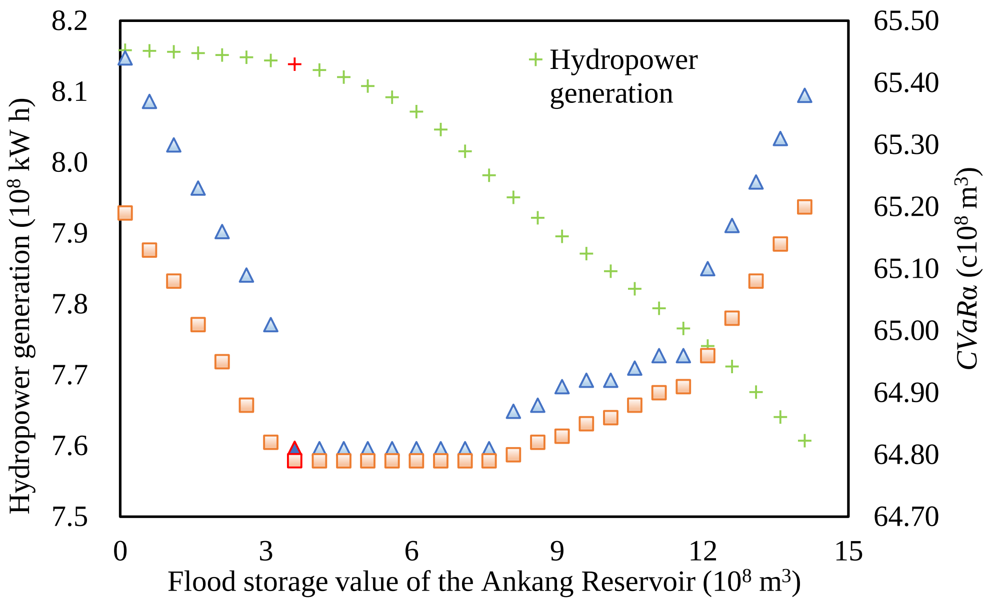
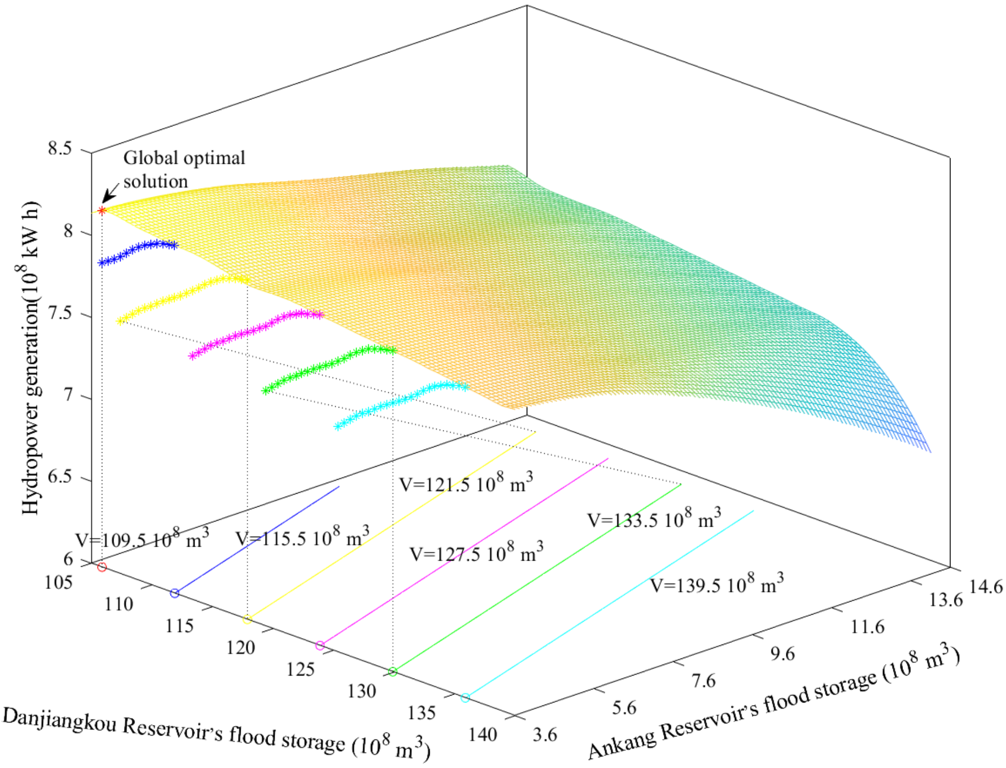


Fig. 10. Optimal result when the total flood storage value is fixed at 11.36 billion m^3.



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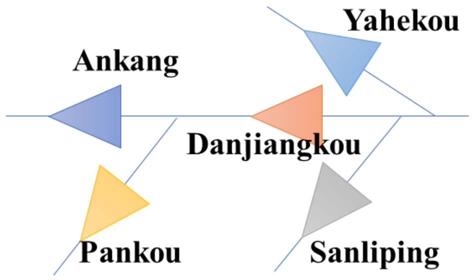

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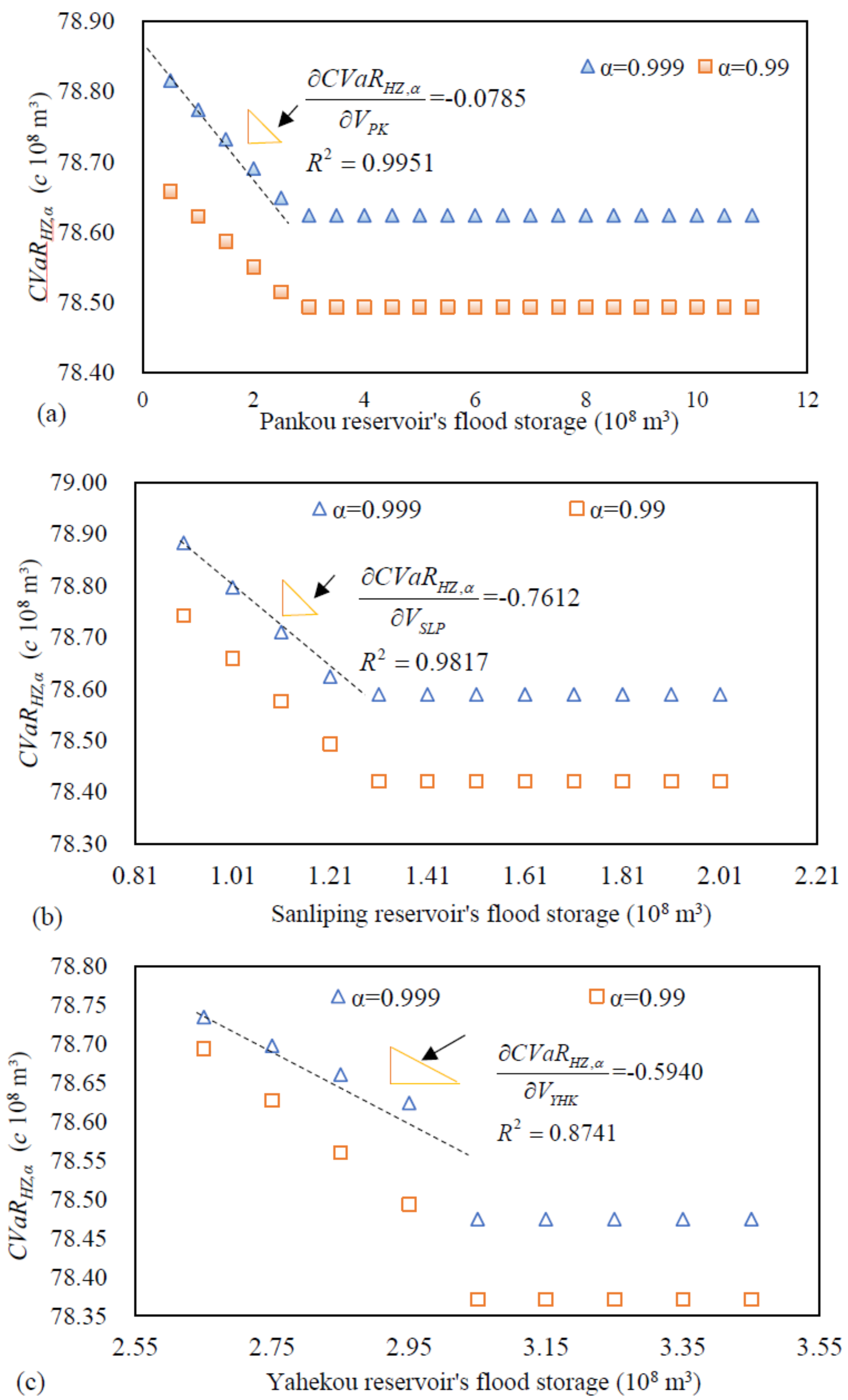

Fig. B2. Results of relationship between the flood damage assessment index $CVaR_{\alpha}$ value and Pankou/Sanliping/Yahekou reservoir's flood storage value.

Declaration of interests

☒The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Author statement

Zhang Xiaoqi: Conceptualization, Methodology, Software Writing- Original Draft **Liu Pan:** Conceptualization, Supervision, Writing- Review & Editing **Feng Maoyuan:** Methodology, Validation, Writing- Review & Editing **Xu Chong-Yu:** Supervision, Writing- Review & Editing **Cheng Lei:** Validation, Writing- Review & Editing **Gong Yu:** Data Curation, Writing- Review & Editing